# Labor Market Power and the Pro-competitive Gains from Trade*

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**Abstract**

We build into a model of trade with market power in product markets and strategic complementarities in price-setting labor market power, strategic complementarities in wage-setting, and a rich structure of product and labor markets to study the pro-competitive gains from trade. We combine our model with confidential firm-level data from Australia to quantify the welfare and efficiency gains of higher exposure to international trade. We find that closing the economy to trade has significant and negative effects on welfare and aggregate labor productivity. In a perfectly competitive economy, where there is no firm market power, welfare losses are only 40% of the total effect which indicates that imperfect competition is the main driver behind these losses. Contrary to common belief, these losses are amplified by the presence of labor market power, which accounts for almost half of the total effect. We find that, at the aggregate level, the pro-competitive effect is well approximated by models of firm market power that leave aside strategic complementarities both in price- and wage-setting. However, these models hide how the losses materialize. Our results highlight the importance of market structure, firm competition, and strategic complementarities for the quantification of the gains from trade liberalizations.

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# 1 Introduction

An old idea in international trade is that a more open economy can alleviate distortions caused by market power in product markets. The conventional view sustains that as an economy opens up to trade, local producers are more exposed to higher competition from foreign firms. Small and less productive firms shrink and exit the market, while larger and more productive firms concentrate local resources and expand to new markets. This pattern of reallocation has been documented, for example, in Pavcnik (2002), Trefler (2004), and Mayer et al. (2014). The higher levels of competition alter the underlying distribution of markups in the economy, and the resulting reallocation of resources places inputs where their value is higher, increasing aggregate productivity (Holmes et al., 2014; Edmond et al., 2015; Arkolakis et al., 2018). Because these effects result from exposure to greater competition, the literature has referred to this mechanism as the pro-competitive gains from trade.

On the other hand, recent evidence from factor markets suggest firms may also exert market power over the local factors of production. Empirical research has estimated positive markdowns[1] on wages for the vast majority of manufacturing plants in the US (Hershbein et al., 2019; Kirov and Traina, 2021).[2] In this context, the higher concentration of local resources that results from exposing the economy to trade has consequences for the prices of these factors of production due to firm market power. For example, Felix (2021) finds that after a trade liberalization, in labor markets where firms face a more substantial reduction in the import tariff of their product, there is a stronger increase in markdowns due to an increase in market concentration. However, it remains an open question how this effect on relative costs across firms feeds back into their product market power, and more important whether these forces affect the level and distribution of the welfare gains from trade.

Our goal is to use producer-level data to quantify the overall effects of trade on productivity and welfare when producers exert market power in both product and labor markets. We study these pro-competitive effects in a quantitative trade model with endogenously variable markups and markdowns. For this, we build a general equilibrium model of trade with market power in product and labor markets, strategic complementarities in price- and wage-setting, and a rich structure of product and labor markets.

The model combines features of Atkeson and Burstein (2008) and Berger et al. (2022a). In the model, competition in product markets is as in Atkeson and Burstein (2008), where in any given market, a small number of producers engage in oligopolistic competition. Firms are non-atomistic within product markets and compete strategically for consumers internalizing how they expect other firms to respond to their production and price policies. Different from this paper is our treatment of the labor market. First, we extend Atkeson and Burstein (2008) by incorporating labor supply decisions endogeneizing the total supply of factor of production in the economy. Second, we depart from the assumption of a perfectly competitive country-level labor market. Instead, within the country there are multiple local labor markets which are perceived as imperfect substitutes from the point of view of the household. Competition in labor markets is as in Berger et al. (2022a) where in any given labor market, there is a small number of producers who engage in oligopsonistic competition. Firms are non-atomistic within labor markets and compete strategically

[1] The firm's markdown is defined as the gap between its marginal revenue product of labor and the wage it pays its workers. In perfectly competitive markets, this gap is zero. On the contrary, when the firm exerts market power, this gap is positive, reflecting the ability of the firm to pay wages below its marginal revenue product of labor.

[2] Likewise, a negative relationship has been documented between the buyer share of a firm sales and the price it pays for inputs consistent with buyer power on input trade (Morlacco, 2020).

for workers internalizing how they expect other firms to respond to their hiring and wage policies. We accommodate this structure using a nested constant elasticity of substitution (CES) product-demand and labor-supply system that under imperfect competition generates both variable markups and markdowns.

We express the model in relative changes and show we can identify the effect on welfare and aggregate productivity of closing the economy to trade using data only from the domestic country in the trade equilibrium. Our counterfactuals are performed with knowledge of a few parameters but are intensive in data requirements. In particular, it requires knowledge about: (i) the boundaries of each product and labor market, (ii) the assignment of firms to these markets, (iii) the equilibrium distribution within each market of firm's domestic sales, total payroll and employment, and (iv) the elasticities of substitutions within and across product and labor markets. The first three elements allow us to compute firm-level market shares, the key input in our equilibrium conditions, and when combined with information on the elasticities of substitutions they determine the equilibrium distribution of firm's market power. This type of information is hardly available for a single country, less for a set of countries. My identification result circumvents the challenge of obtaining such granular data for several countries by establishing that, when we focus on the counterfactual in which the economy does not participate in international trade, one only needs information for the domestic economy.[3]

We quantify the model using a new data set containing detailed confidential firm-level data from Australia. The data comes from the Business Longitudinal Analysis Data Environment (BLADE), an economic data tool that combines tax, trade, and intellectual property data on all active businesses in Australia sourced from surveys conducted by the Australian Bureau of Statistics Business Register and the Australian Taxation Office. We use the information in BLADE on firm's domestic and export sales, total payroll, employment, and the firm's industry classification and location for all manufacturing plants actively producing in the fiscal year of 2018/2019 to discipline the model along the four key elements described before.

We partition the economy into a finite number of product and labor markets and use the information on a firm's location and industry classification to define the boundaries of each market. Several studies have shown that labor markets are local because workers find it costly to search for jobs in locations far from where they live. For example, Manning and Petrongolo (2017) estimates that the attractiveness of jobs decays sharply with distance. At the same time, workers require specific skills to produce a particular good, and therefore workers cannot move easily across different industries either (Neal, 1995; Parent, 2000). To capture these features, we define labor markets based on sector-region pairs using a unique characteristic of our data. BLADE matches its information on firm-level tax data with the Australian Statistical Geography Standard, a classification of Australia into statistical areas designed to represent labor markets.[4] We augment this definition of labor markets by interacting each statistical area with a 3-digit manufacturing industry of the Australian and New Zealand Standard Industry classification from 2006 (ANZSIC06), giving us approximately 3500 local labor markets. Firm's output, on the other hand, is considerably more mobile than labor, so we define product markets based only on different manufacturing industries as in Edmond et al. (2015) and Gaubert and Itskhoki (2021).[5] In total, we divide the economy into 143 distinct

[3]To study a larger set of counterfactuals with this approach, for example cases with more or less degrees of openness to trade, we would require to observe this information not only for the domestic economy but also for each of its trade partners.

[4]Each hierarchy is designed to measure specific economic outcomes with the Statistical Areas - Level 4 (SA4) classification designed to represent labor markets. Statistical Areas - Level 4 (SA4) were designed by taking into account information on where people live (labor demand) and where people work (labor supply).

[5]With our data we cannot incorporate regional components to our definition of product markets since we lack

product markets, each a 4-digit manufacturing industry of ANZSIC06. Finally, we calibrate the elasticities of substitution within and across these markets using numbers from the literature and information about the sales to payroll ratio in BLADE. We show that our results are robust to a range of parameters in the literature that relies on the nested CES structure for product-demand and labor-supply.

We find that closing the economy to international trade generates a 2% decrease in welfare. These losses result from of a lower amount of aggregate consumption and a higher number of hours worked, which tries to compensate for the fall in output due to the absence of trade. Although the aggregate effect is small, the imperfectly competitive nature of product and labor markets accounts for the majority of these losses. In a perfectly competitive economy, where the allocation of resources is efficient, and there is no firm market power, the welfare losses of moving the economy to autarky are only 40% of the total effect.

To understand the forces at play that drive the losses in welfare and efficiency, we quantify the effects of trade across different classes of models nested in our specification. We start by looking at versions of the model where firms behave strategically in only one market while in the other we impose perfect competition. In these cases, the model reduces to either a variant of Edmond et al. (2015) with many local labor markets or an open economy version of Berger et al. (2022a). We find in these cases that the estimated welfare and efficiency effects are significantly smaller. On average this models under predict the welfare gains by 40% and can miss by almost 90% the affect on aggregate productivity. While both models tend to miss most of the effect of trade their relative performance depends on the value of the elasticities of substitution within product and labor markets.

Furthermore, we find that the effects of imperfect competition and strategic complementarities in product and labor markets interact with each other. As we mentioned before, neither of these models can reproduce the total effect of trade and combined they still miss as much as 25%, indicating that the distortions caused by firms' markdowns tend to amplify those generated by their markups, and vice versa. These results confirm that studying product and labor market power in a unified framework is important to understand the overall effects of trade and that these features are quantitatively important.

Finally, we remove strategic complementarities in price- and wage-setting from the model but still allow firms to exert market power in product and labor markets. In this case, firms are monopolists (monopsonists) in product (labor) markets and charge a constant markup (markdown) as in Jha and Rodriguez-Lopez (2021). We find that, on average, welfare losses are well approximated by a model with constant levels of market power, and so the pro-competitive gains are in this sense *elusive* as defined by Arkolakis et al. (2018).[6] In several important economic models, the unconditional markup distribution is invariant to the level of trade costs. Therefore, using changes in the moments of the markup distribution to infer the effects of trade on welfare or aggregate productivity can lead to wrong conclusions. While there is not an equivalent treatment in the literature for the relationship between trade and the distribution of markdowns, we show that there are knife-edge cases in our model where the unconditional markdown distribution is also invariant to the level of trade costs. In fact, our quantitative results shows an almost invariant distribution for markdowns and welfare losses from closing the economy to international trade.

Quantifying potential welfare gains and costs from trade has become increasingly important over

information on goods flow within Australia.

[6]The losses in aggregate productivity are caused by the dispersion in firm's market power within product and labor markets. In the limit where firm market power is constant, there is no misallocation of resources. The equilibrium of this model, however, is not efficient since market power operates as an aggregate wedge inducing the household to work more than would be efficient.

the years. Our paper relates to an extensive literature that evaluates trade policy and is mostly related to studies that quantify the pro-competitive gains from trade.

Since the seminal work by Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), many papers have tried to understand what drives the misallocation of resources and what can be done about it. We focus on the role of endogenous firm market power variation as a source of misallocation and the effectiveness of international trade addressing it as in Epifani and Gancia (2011), Holmes et al. (2014), Edmond et al. (2015), Arkolakis et al. (2018), Jha and Rodriguez-Lopez (2021), Egger et al. (2021), Felix (2021), and MacKenzie (2021). Our modeling approach to output market power follows closely Atkeson and Burstein (2008), and therefore our work is closely related to Edmond et al. (2015). Nonetheless, our paper still differs from these studies in several dimensions. First, we explicitly consider endogenous labor supply decisions, and so trade not only affects the composition of labor across markets but also the total amount of factor of production in the economy. Secondly, we depart from the assumption of a perfectly competitive labor market. Within the country, multiple local labor markets are populated by heterogeneous firms exerting market power over the labor input. We show how accounting for the interrelation between imperfect competition in product and labor markets amplifies the welfare and efficiency gains from trade. Thirdly, we allow for arbitrary boundaries for product and labor market. Hence, the model accommodates a salient feature of the data in which a product is produced by firms operating in different labor markets and, at the same time, within one labor market firms produced different products. Fourthly, the way in which we take the model to the data is very different compared to these studies. We solve our two-country and multi-market model in changes relative to a base year, allowing us to perform counterfactuals without relying on estimates of unobserved structural parameters, like fundamental productivity, nor imposing assumptions about the rest of the world. Finally, our identification strategy for the gains from trade allows us to handle fixed cost of exporting without the complications of problems with discrete choice or multiplicity of equilibrium that is common in environments that rely on imperfect competition à la Atkeson and Burstein (2008).

Our paper is also related to the literature on imperfect competition in labor markets (Manning, 2011; Card et al., 2018; Manning, 2021; Berger et al., 2022a) and the subsequent papers that address the question of the optimal policy response (Berger et al., 2022b; Mousavi, 2021; Trottner, 2022). All of this papers have looked at the effects of labor market power within closed-economy models and therefore we contribute to this literature introducing international trade. Furthermore, the potential policy solutions that the literature has given to labor market power has focus on instruments that use prices, .e.g., minimum wage or taxes on labor. We show that opening to trade helps to address this problem by differently exposing markets to foreign competition.

The remainder of the paper proceeds as follows. Section 2 presents the model. Section 3 gives an overview of the data, and Section 4 explains how we use that data to quantify the model. In Section 5, we present our main results on the pro-competitive gains from trade and robustness to some of our parameter choices. Finally, in Section 6, we conclude.

## 2 Theoretical Framework

The world consist of two symmetric countries, Home and Foreign.[7] We assume a static environment with a single factor of production, labor, that is immobile between countries. We focus on describing

[7]We impose this assumption here, but, in the quantitative section, we work with two asymmetric countries.

the Home country in detail. We indicate Foreign variables with an asterisk.[8]

Each economy is partitioned into a finite (but large) number of product and labor markets. We use $\mathcal{K}$ and $\mathcal{L}$ to denote the set of product and labor markets, respectively.[9] At the same time, the economy is populated by a fixed and finite number of firms. These firms are denoted with $i$ and $j$, and we use $\mathcal{M}$ to refer to the set of firms. Each firm $i \in \mathcal{M}$ hires workers from a specific labor market $l \in \mathcal{L}$ to produce a specific type of good $k \in \mathcal{K}$. We assume that firms have market power both in product and labor market.

We define two functions $\kappa : \mathcal{M} \cup \mathcal{M}^* \to \mathcal{K}$ and $\ell : \mathcal{M} \to \mathcal{L}$, that map firms into product and labor markets, respectively. In the model, these functions work as accounting devices that help to keep track of which firms operate in each market, and therefore determine the set of competitors for a firm. However, at their core they encode the definition of product and labor markets. At this point, these functions are arbitrary and flexible enough to accommodate different definitions of what constitutes a market.[10]

Finally, there is unit mass of identical households that supply labor to all firms and consumes the final good which is an aggregate basket of all the goods produced in the Home and Foreign country.

## 2.1 Household

**Preferences.** The household chooses the number of workers to supply to each firm $n_i$ and consumption of the final good $C$ to maximize their value of utility. Preferences are described by the utility function

$$\mathcal{U} = \log C - \bar{\varphi}^{\frac{1}{\varphi}} \frac{N^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}, \tag{1}$$

where $N$ is the aggregate labor supply index and $\bar{\varphi} > 0$ and $\varphi > 0$ are two parameters capturing household disutility from labor and the Frisch elasticity of labor supply, respectively. We assume that the aggregate labor supply index follows the nested CES structure of Berger et al. (2022a) and is given by:

$$N = \left[\sum_{\ell \in \mathcal{L}} \upsilon_{n,l}^{-\frac{1}{\theta}} N_\ell^{\frac{\theta+1}{\theta}}\right]^{\frac{\theta}{\theta+1}}, \qquad N_\ell = \left[\sum_{\{i \in \mathcal{M}: \ell(i)=l\}} \xi_{n,i}^{-\frac{1}{\eta}} n_i^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}, \qquad \eta \geq \theta > 0, \tag{2}$$

where the inner nest aggregates employment across firms within a particular labor market $l \in \mathcal{L}$ while the outer nest aggregates this market level employment indexes across different labor markets. The two key parameters in equation (2) are $\eta$ and $\theta$ which represent the elasticity of substitution across different employers within a market, and the elasticity of substitution across different labor

[8] The description of the environment follows closely Edmond et al. (2015) with the distinction that labor markets are model as in Berger et al. (2022a).

[9] In the Foreign country, the set of product and labor markets are $\mathcal{K}^*$ and $\mathcal{L}^*$

[10] In Atkeson and Burstein (2008) and Edmond et al. (2015) firms produce for different sectors and hire workers from a unique labor market. This corresponds to $\mathcal{K}$ representing the different sectors in the economy, $\kappa$ identifies the sector of the firm, and $|\mathcal{L}| = 1$. In Berger et al. (2022a), firms operate in different labor markets and produce varieties of a unique type of good. This correspond to $\mathcal{L}$ representing the different labor markets in the economy, $\ell$ identifies the labor market where the firm operates, and $|\mathcal{K}| = 1$. Finally, the literature that has worked with models of imperfect competition in product and labor markets impose the restrictive assumption that each product market is also a labor market. (Deb et al., 2022; Trottner, 2022). In this case, $\mathcal{K} = \mathcal{L}$. We want to think of cases outside these extremes where, for example, two firms from different industries are located in the same region, so although they do not compete for the same consumers they do compete for the same pool of workers.

markets, respectively. Finally, the parameters $\{\xi_{n,i}\}_{i\in\mathcal{M}}$ ($\{\upsilon_{n,l}\}_{l\in\mathcal{L}}$) are non-negative labor supply shifters that reflect the preferences of the household to work in one over another firm (labor market), such that,

$$\sum_{\{i\in\mathcal{M}:\ell(i)=l\}} \xi_{n,i} = 1, \quad \text{and} \quad \sum_{l\in\mathcal{L}} \upsilon_{n,l} = 1.$$

The preferences over labor supply in equation (2) imply that jobs are differentiated from the perspective of the workers, this constitutes the source of labor market power in the model. Berger et al. (2022a) show that this type of preferences can be micro-funded in an environment with heterogeneous workers making independent decisions on where to work providing an exact map between the parameters $\eta$ and $\theta$ and the distribution of relative net costs to individuals of moving between and across markets.

**Utility Maximization.** Consumers maximize utility $\mathcal{U}$ subject to the budget constraint,

$$C = \sum_{i\in\mathcal{M}} w_i n_i + \Pi,$$

where $w_i$ is the wage offered by employer $i$ and $\Pi$ are aggregate profits. The expression for the budget constraint makes explicit that we use final consumption in Home as the numeraire.

Solving the household's problem, we obtain that the labor supply face by employer $i$ in market $l$ is given by:

$$n_i = \xi_{n,i}\upsilon_{n,\ell(i)} \left(\frac{w_i}{W_{\ell(i)}}\right)^{\eta} \left(\frac{W_{\ell(i)}}{W}\right)^{\theta} N, \tag{3}$$

with $W_{\ell(i)}$ and $W$ the market and aggregate wage index, where $\ell(i) \in \mathcal{L}$ indicates the labor market where firm $i$ operates. These indexes are defined as the numbers that satisfy

$$W_l N_l = \sum_{\{i\in\mathcal{M}:\ell(i)=l\}} w_i n_i, \qquad WN = \sum_{l\in\mathcal{L}} W_l N_l.$$

Together with optimality condition (3) these definitions imply,

$$W_l^{1+\eta} = \sum_{\{i\in\mathcal{M}:\ell(i)=l\}} \xi_{n,i} w_i^{1+\eta}, \qquad W^{1+\theta} = \sum_{l\in\mathcal{L}} \upsilon_{n,l} W_l^{1+\theta}. \tag{4}$$

Finally, consumption of final good $C$ (the numeraire) and the aggregate labor supply index $N$ satisfy the standard consumption-labor trade-off equation,

$$(\bar{\varphi}N)^{\frac{1}{\varphi}} C = W. \tag{5}$$

## 2.2 Final Good Producers

**Technology.** Perfectly competitive firms in each country produce an homogeneous final consumption good $Y$ using a bundle of inputs $Y_k$ from each market $k \in \mathcal{K}$,

$$Y = \left[\sum_{k\in\mathcal{K}} \upsilon_{y,k}^{\frac{1}{\epsilon}} Y_k^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{6}$$

where $\epsilon > 1$ is the elasticity of substitution across product markets $k \in \mathcal{K}$. Each product market consists of a finite number of domestic and foreign intermediate producers. In market $k$, output is produced using $M_k \in \mathbb{N}$ domestic intermediate inputs and potentially an additional $M_k^* \in \mathbb{N}$ imported intermediate inputs,

$$Y_k = \left[ \sum_{\{i \in \mathcal{K}: \kappa(i)=k\}} \xi_{Hy,i}^{\frac{1}{\sigma}} y_{H,i}^{\frac{\sigma-1}{\sigma}} + \sum_{\{i \in \mathcal{M}^*: \kappa(i)=k\}} \xi_{Fy,i}^{\frac{1}{\sigma}} y_{F,i}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{7}$$

where $y_{c,i}$ denotes the output of firm $i$ located in country $c \in \{H, F\}$ and $\sigma \geq \epsilon$ is the elasticity of substitution across goods within a particular market $k$. The parameters $\{\xi_{cy,i}\}_{i \in \mathcal{M} \cup \mathcal{M}^*, c \in \{H,F\}}$ and $\{\upsilon_{y,k}\}_{k \in \mathcal{K}}$ are non-negative demand shifters that reflect the preferences of the final good producer for inputs of one over another firm producing goods for different markets, such that,

$$\sum_{\{i \in \mathcal{M}: \kappa(i)=k\}} \xi_{Hy,i} + \sum_{\{i \in \mathcal{M}^*: \kappa(i)=k\}} \xi_{Fy,i} = 1,$$

and

$$\sum_{k \in \mathcal{K}} \upsilon_{y,k} = 1.$$

**Demand for Intermediate Inputs.** Final good producers buy intermediate goods from Home producers at prices $p_{H,i}$ and from Foreign producers at prices $p_{F,i}$. A final good producer chooses intermediate inputs $y_{H,i}$ and $y_{F,i}$ to maximize profits,

$$Y - \sum_{k \in \mathcal{K}} \left( \sum_{\{i \in \mathcal{M}: \kappa(i)=k\}} p_{H,i} y_{H,i} + \sum_{\{i \in \mathcal{M}^*: \kappa(i)=k\}} p_{F,i} y_{F,i} \right),$$

subject to (6) and (7). The solution to this problem gives the demand functions for domestic and foreign inputs,

$$y_{H,i} = \xi_{Hy,i} \upsilon_{y,\kappa(i)} \left( \frac{p_{H,i}}{P_{\kappa(i)}} \right)^{-\sigma} \left( \frac{P_{\kappa(i)}}{P} \right)^{-\epsilon} Y, \quad y_{F,i} = \xi_{Fy,i} \upsilon_{y,\kappa(i)} \left( \frac{p_{F,i}}{P_{\kappa(i)}} \right)^{-\sigma} \left( \frac{P_{\kappa(i)}}{P} \right)^{-\epsilon} Y. \tag{8}$$

with $P_{\kappa(i)}$ and $P$ the market and aggregate price index, and $\kappa(i) \in \mathcal{K}$ indicating the product market where firm $i$ operates. These indexes follow the standard formulas with

$$P = \left[ \sum_{k \in \mathcal{K}} \upsilon_{y,k} P_k^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}, \tag{9}$$

and

$$P_k = \left[ \sum_{\{i \in \mathcal{M}: \kappa(i)=k\}} \xi_{Hy,i} p_{H,i}^{1-\sigma} + \sum_{\{i \in \mathcal{M}^*: \kappa(i)=k\}} \xi_{Fy,i} p_{F,i}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{10}$$

where we sum over all the firms that operates in market $k$, i.e. all the firms for which $\kappa(i) = k$ is true.

## 2.3 Intermediate Good Producers

**Technology.** It is defined by three properties: (i) the production technology; (ii) the identity of the labor market from where the firm hires her workers; and (iii) the identity of the product market where the firm sells her good. The first component is standard and determines how much output the firm can produce for a given amount of inputs. The other two determine the amount of competition the firm faces for workers and consumers.

The production technology of intermediate good producer $i$ is a constant returns to scale production function that has labor as the only input. Firm $i$ hires workers in labor market $\ell(i) \in \mathcal{L}$ to produce goods for product market $\kappa(i) \in \mathcal{K}$. The production function is given by,

$$y_i = z_i n_i, \tag{11}$$

where producer-level productivity $z_i$ is the realization from the country specific distribution $F(z)$.[11]

**Trade Costs.** The output of firm $i$ can be traded under two types of trade costs. We assume there is a fixed cost of exporting that requires $f_k \geq 0$ units of the labor supply index for any firm that wishes to export any of its output to the other country.[12] We also assume there is an iceberg type cost of exporting $\tau_k \geq 1$. Let $y_{H,i}$ denote the amount sold by a Home intermediate good producer to the Home final good producer, and similarly let $y^*_{H,i}$ denote the amount sold by a Home intermediate good producer to the Foreign final good producers. The resource constraint for Home intermediate good producers is,

$$y_i = y_{H,i} + \tau_{\kappa(i)} y^*_{H,i}. \tag{12}$$

**Market Structure.** We assume that the intermediate good producing firms are engaged in imperfect competition both in the product and labor market playing a static game of quantity competition. Specifically, each firm $i$ chooses its quantity $y_{H,i}$ and $y^*_{H,i}$ for the home and foreign market, respectively, taking as given the quantities chosen by the other firms in the economy, recognizing that sectoral prices vary when the firm changes its quantity. Simultaneously, each firm $i$ chooses it demand for labor $n_i$ taking as given the number of workers hired by the other firms operating in the same labor market $\ell(i)$, recognizing that market level wages vary when the firm changes its demand for labor. Although we model a finite number of product and labor markets, we assume that firms are atomistic with respect to the aggregate, and therefore take the aggregate price and aggregate wage as given.

We decide not to model the entry decision to the domestic market, and instead assume that the number of domestic firms within each product and labor market are exogenously given. Modeling entry is challenging because a decision to enter into a particular product and labor market needs to consider: i) which firms are already selling their output in product market $k$; and ii) which firms are hiring labor from labor market $l$. Both of these considerations opens the possibility of having multiplicity of equilibria, since the entry decision of the firm depends on the entry decisions of all the other firms.

[11] Because we have a finite number of firms within each sector, market-level averages do not coincide with moments from the distribution $F(z)$. Therefore, the productivity parameters $z_i$ are a source of granular comparative advantages between the two countries as in Gaubert and Itskhoki (2021).

[12] The choice of the units of the fixed costs are not innocuous when we look at welfare and aggregate productivity (see for example Baqaee and Farhi, 2020; Gutierrez, 2022). We denote fixed costs in units of aggregate labor supply index $N$, as opposed to $n_i$ for example, to avoid endowing the firm with market power over the fixed resources used for exporting.

However, the total number of firms in a product market, both domestic and foreign, that sell positive amounts of their goods in each country is determined endogenously in equilibrium - firms will choose to export if it is profitable for them to do so.[13]

**The problem of the firm.** A Home intermediate good producer chooses quantities $y_{H,i}$ and $y_{F,i}$ for the Home and Foreign market, labor units $n_i$, and whether to export or not $\phi_i \in \{0,1\}$ in order to maximize profits,

$$\pi_i \equiv \max_{y_{H,i},y_{F,i},n_i} \left[ y_{H,i} p_{H,i} + \phi_i y^*_{H,i} p^*_{H,i} - w_i n_i - \phi_i f_k W \right], \tag{13}$$

subject to the inverse labor supply curve, equations (3) and (4), the inverse product demand curve, equations (8) and (10), and the technology of the firm, equations (11) and (12), and taking as given the amount of output being produced by $i$'s competitors in the product market as well as the number of workers being hired by $i$'s competitors in the labor market.

A couple of things related to the characteristics describing a firm are worth emphasizing before describing the solution to the firms problem. The first thing to notice is that the set of competitors a firm faces differ across product and labor markets. Trivially, foreign producer do not hire workers from Home, but the generic assignment functions $\kappa(\cdot)$ and $\ell(\cdot)$ imposed at this point implies that even the domestic competitors of firm $i$ in the product market do not necessarily coincide with those competing against $i$ for workers in the labor market. These partial overlap across the set of competitors of a firm implies that different product markets are interconnected through labor markets, and therefore labor markets will play a role propagating shocks across product markets when one is suddenly more exposed to trade.

The second thing to notice is that labor market power interacts with the technology of the firm creating convexities in the cost function. The total cost a firm has to pay for a target value of output $y$ she wishes to produce equals the amount of workers required to produce that amount, $\frac{y}{z_i}$, times the amount of wages she has to pay to those workers. Labor market power implies that the firm faces the labor supply equation (3), and therefore the amount of wages she has to pay depends on the amount of workers she wants to hire, $w_i\left(\frac{y}{z_i}; \mathbf{n_{-i}}, \mathbf{W}\right)$,

$$c_i(y) = w_i\left(\frac{y}{z_i}; \mathbf{n_{-i}}, \mathbf{W}\right)\frac{y}{z_i}.$$

Therefore, even though firms operate a linear technology, the decision of how much output to produce for the domestic and the foreign economy *cannot* be considered separately: The optimal amount of output produced for each country equates marginal revenue with marginal cost, however the marginal cost is a function of the total amount produced for both markets. Intuitively, if a firm decides to increase its scale of production to sell abroad it has to hire extra workers; but because of the firm's labor market power this can only be done by offering higher wages than its competitors. The higher wage, however, does not only apply to the marginal workers hired by the firm but to all previously hired workers which are used to produce goods for the domestic economy.[14] This feature

[13] The discrete decision of whether to export or not also brings the possibility of multiple equilibria which compromises any counterfactual analysis of the model. To overcome this issue, in the quantitative section, we treat the observe data as an equilibrium of the model and then look at the counterfactual equilibrium with no international trade which is unique given our assumptions on firm entry.

[14] This implies that absent fixed cost of exporting, if a firm were able to choose between operating as a unified firm selling to the foreign and domestic economy or to split into two firms, it would choose the latter. With positive cost of exporting the decision will depend on whether the exporting-establishment is able to cover the fixed cost by itself.

of the model distinguish our framework from previous papers that study firm's export decisions, where perfectly competitive labor markets and linear technologies (i.e. constant marginal cost) allows them to treat the two markets separately (see for example Melitz, 2003; Chaney, 2008).[15]

Despite this extra difficulty in the firm's problem, optimal prices still satisfy standard formulas. Profit-maximization implies that a firm's optimal price is a markup over its marginal cost $mc_i$,

$$p_{H,i} = \mu_i^y mc_i, \qquad p_{H,i}^* = \mu_i^{y^*} \tau_{\kappa(i)} mc_i, \tag{14}$$

where the markups satisfy the usual Lerner formula,

$$\mu_i^y = \frac{\gamma_i^y}{\gamma_i^y - 1}, \qquad \mu_i^{y^*} = \frac{\gamma_i^{y^*}}{\gamma_i^{y^*} - 1}, \tag{15}$$

with $\gamma_i^y \equiv -\left[\frac{\partial \log p_{H,i}}{\partial \log y_{H,i}}\right]^{-1}$ and $\gamma_i^{y^*} \equiv -\left[\frac{\partial \log p_i^*}{\partial \log y_i^*}\right]^{-1}$ being the price elasticity of demand.

Profit-maximizing wages equal a markdown over a firm's marginal revenue product of labor $mrpl_i$,

$$w_i = \mu_i^n mrpl_i, \tag{16}$$

where the markdown depends inversely on the labor supply elasticity faced by the firm $\gamma_i^n \equiv \left[\frac{\partial \log w_i}{\partial \log n_i}\right]^{-1}$,

$$\mu_i^n = \frac{\gamma_i^n}{1 + \gamma_i^n}. \tag{17}$$

Under our specification of preferences, these elasticities have closed-form expressions and are given by,

$$\begin{aligned} \gamma_i^y &= \left[\frac{1}{\sigma} + \left(\frac{1}{\epsilon} - \frac{1}{\sigma}\right) s_{Hi}^y\right]^{-1}, \\ \gamma_i^{y^*} &= \left[\frac{1}{\sigma} + \left(\frac{1}{\epsilon} - \frac{1}{\sigma}\right) s_{Hi}^{y^*}\right]^{-1}, \\ \gamma_i^n &= \left[\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n\right]^{-1}, \end{aligned} \tag{18}$$

where $s_{Hi}^y$ and $s_{Hi}^{y^*}$ are firm's $i$ sales-share in the Home and Foreign economy, respectively, and $s_i^n$ is the firm's payroll-share. Within a product (labor) market, firms with relatively high market shares have low demand (supply) elasticity and high markups (markdowns). Equations (14) to (18) are the open economy counterparts with oligopsonistic labor markets of Edmond et al. (2015) and Berger et al. (2022a).

The marginal cost for a firm is given by $mc_i = \frac{1}{\mu_i^n} \frac{w_i}{z_i}$, hence wages and price are related through the pricing equations,

$$p_{H,i} = \frac{\mu_i^y}{\mu_i^n} \frac{w_i}{z_i}, \qquad p_{H,i}^* = \frac{\mu_i^{y^*}}{\mu_i^n} \tau_{\kappa(i)} \frac{w_i}{z_i}. \tag{19}$$

Together, equations (18) and (19) highlight that prices and wages are jointly determined by the firm's choice level of employment and aggregate conditions in the labor and product market a point made also in Trottner (2022).

[15] We go back to this point later in the text when we study the effect of lower trade costs. Because of this curvature in the marginal costs, firms do not increase their scale of production as much as they would do absent labor market power.

## 2.4 Equilibrium

In a decentralized equilibrium, households in each country maximize utility by choosing how much to consume of the final good and how many workers to supply to each firm, firms make output and employment decisions to maximize their profits, all the product and labor markets clear, and trade between Home and Foreign is balanced. Definition 2.1 formally defines the equilibrium, summarized by equations (2) to (20).

**Definition 2.1.** An equilibrium is an allocation $\{\{y_{c,i}, y^*_{c,i}\}_{c\in\{H,F\}}, n_i, n^*_i\}_{i\in\mathcal{M}\cup\mathcal{M}^*}, Y, Y^*, N, N^*\}$, prices $\{\{p_{c,i}, p^*_{c,i}\}_{c\in\{H,F\}}, w_i, w^*_i\}_{i\in\mathcal{M}\cup\mathcal{M}^*}$, and export decisions $\{\phi_i, \phi^*_i\}_{i\in\mathcal{M}\cup\mathcal{M}^*}$ such that:

1. Given prices $\{\{p_{c,i}, p^*_{c,i}\}_{c\in\{H,F\}}, w_i, w^*_i\}_{i\in\mathcal{M}\cup\mathcal{M}^*}$, the quantities $\{\{n_i, n^*_i\}_{i\in\mathcal{M}\cup\mathcal{M}^*}, Y, Y^*, N, N^*\}$ maximize the utility of the domestic and foreign household.

2. Given prices $\{\{p_{c,i}, p^*_{c,i}\}_{c\in\{H,F\}}\}_{i\in\mathcal{M}\cup\mathcal{M}^*}$, the quantities $\{\{y_{c,i}, y^*_{c,i}\}_{c\in\{H,F\},i\in\mathcal{M}\cup\mathcal{M}^*}, Y, Y^*\}$ maximize the profit of the final good producer in the domestic and foreign country.

3. Prices $\{\{p_{c,i}, p^*_{c,i}\}_{c\in\{H,F\}}, w_i, w^*_i\}_{i\in\mathcal{M}\cup\mathcal{M}^*}$, quantities $\{\{y_{c,i}, y^*_{c,i}\}_{c\in\{H,F\}}, n_i, n^*_i\}_{i\in\mathcal{M}\cup\mathcal{M}^*}$, and export decisions $\{\phi_i, \phi^*_i\}_{i\in\mathcal{M}\cup\mathcal{M}^*}$ maximize profits of the intermediate good producer in the domestic and foreign country.

4. Trade is balanced between countries, and therefore prices $\{\{p_{c,i}, p^*_{c,i}\}_{c\in\{H,F\}}, w_i, w^*_i\}_{i\in\mathcal{M}\cup\mathcal{M}^*}$ and quantities $\{y_{c,i}, y^*_{c,i}\}_{c\in\{H,F\},i\in\mathcal{M}\cup\mathcal{M}^*}$ satisfy,

$$\sum_{i\in\mathcal{M}} p^*_{H,i} y^*_{H,i} = \sum_{i\in\mathcal{M}^*} p_{F,i} y_{F,i}. \tag{20}$$

5. All the product and labor markets clear.

Although, the equilibrium of the model cannot be solved for explicitly, it is insightful to go through the equations that characterize the firm-level market shares since they determine the resulting distribution of resources across firms and are tightly related with the amount of firm-level market power.

With that purpose in mind we define some useful notation moving forward. We let $\{S^y_k\}_{k\in\mathcal{K}}$ to denote product $k$'s share of aggregate sales, and $\{S^n_l\}_{l\in\mathcal{L}}$ to denote labor market $l$'s share of aggregate payrol. These are the market-level counterparts of our firm-level shares. We let $\{\lambda_k\}_{k\in\mathcal{K}}$ denote the share of imports in total sales in product market $k$.

**An Expression for Firm-level Shares.** The size of a firm in a product and in a labor market it is given by the share of the market that the firm captures. These shares summarize how resources are distributed across firms an determine the equilibrium level of firm market power, as it is evident from equations (15), (17), and (18).

In what follows, we impose two assumptions. Assumption 2.1 is a mathematical simplification that implies that the resulting equilibrium allocation does not depend on agents preference from one firm over the other but just on technological features of the firms. We note, however, that the product demand and labor supply shifters cannot be identify separately from the technology component $z_i$. So, alternatively, we can think of $z_i$ as summarizing both technology and preferences. Assumption 2.2 is more consequential since it rules out selection into exporting based on firm's productivity levels.

**Assumption 2.1.** The product demand and labor supply shifters, parameters $\left\{\xi_{cy,i}, \xi^*_{cy,i}, \upsilon_{y,k}, \upsilon^*_{y,k}\right\}$ and $\left\{\xi_{n,i}, \upsilon_{n,k}, \xi^*_{n,i}, \upsilon^*_{n,k}\right\}$, are all equal to one.

**Assumption 2.2.** Trade between Home and Foreign is free.

Under Assumptions 2.1 and 2.2 firm optimality, the production technology of the intermediate good producers, and the labor supply and product demand equations that each firm faces we establish the following property of the equilibrium firm-level shares.[16]

**Proposition 2.1.** For given values of market-level sales and payroll shares, $\{S^y_k\}_{k\in\mathcal{K}}$ and $\{S^n_l\}_{l\in\mathcal{L}}$, and import expenditure shares$\{\lambda_k\}_{k\in\mathcal{K}}$, the equilibrium values of the firm-level shares $\{s^n_i, s^y_{H,i}\}_{i\in\mathcal{M}}$ solve the following system of equations. Payroll shares are given by:

$$s^n_i = \frac{\left(\frac{\mu^n\left(s^n_i\right)}{\mu^y\left(s^y_i\left(1-\lambda_{\kappa(i)}\right)\right)}\right)^{\frac{(1+\eta)\sigma}{\eta+\sigma}} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(S^y_{\kappa(i)}\right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\ell(j)=\ell(i)\}}\left(\frac{\mu^n\left(s^n_j\right)}{\mu^y\left(s^y_j\left(1-\lambda_{\kappa(j)}\right)\right)}\right)^{\frac{(1+\eta)\sigma}{\eta+\sigma}} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(S^y_{\kappa(j)}\right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}}, \tag{21}$$

domestic sales share are $s^y_{Hi} = s^y_i\left(1-\lambda_{\kappa(i)}\right)$ with,

$$s^y_i = \frac{\left(\frac{\mu^n\left(s^n_i\right)}{\mu^y\left(s^y_i\left(1-\lambda_{\kappa(i)}\right)\right)}\right)^{\frac{(\sigma-1)\eta}{\eta+\sigma}} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(S^n_{\ell(i)}\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=\kappa(i)\}}\left(\frac{\mu^n\left(s^n_j\right)}{\mu^y\left(s^y_j\left(1-\lambda_{\kappa(j)}\right)\right)}\right)^{\frac{(\sigma-1)\eta}{\eta+\sigma}} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(S^n_{\ell(j)}\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}, \tag{22}$$

and firm-level market power is given by,

$$\mu^n(s) = \left[\frac{\eta+1}{\eta} + \left(\frac{1}{\theta}-\frac{1}{\eta}\right)s\right]^{-1}, \quad \text{and} \quad \mu^y(s) = \left[\frac{\sigma-1}{\sigma} - \left(\frac{1}{\epsilon}-\frac{1}{\sigma}\right)s\right]^{-1}.$$

*Proof.* See Appendix A.2. □

Proposition 2.1 shows how to compute the firm-level shares. Different from the work of Berger et al. (2022a) and Deb et al. (2022), where firm-level shares are independent of economy aggregates, we do require knowledge of market-level aggregates to compute firm-level shares. For example, payroll shares do not depend on labor market aggregates, since those affect all firms in the market alike. However, they do depend on market level aggregates related to product markets since firms within a labor market produce different types of products, and so their scale of production is related to the market conditions in their respective product markets.[17]

Although Proposition 2.1 does not provide a characterization of the general equilibrium of the model, since the market-level shares are themselves equilibrium objects, it highlights the forces in the model that shape the allocation of resources across firms and the role that market power plays in it.

**Missallocation and Market Power Dispersion.** In this model, dispersion in firm-level market power reduces aggregate productivity, as in the work of Restuccia and Rogerson (2008), Hsieh

[16]The proof can be found in Appendix A.

[17]With costly trade firms in a labor market have different export decisions, and so in this case the level of demand in the Foreign country also affects the distribution of payroll-shares.

and Klenow (2009) ,and Edmond et al. (2015). To understand this effect, notice that total output produced in each product market can be written as

$$Y_k = A_k \times N_k,$$

where $A_k$ is the endogenous level of labor productivity and $N_k$ is the total amount of workers employed in the production of goods for product market $k$.[18] Using the optimality conditions and market clearing it can be show that productivity in market $k$ is given by,

$$A_k = \left( \sum_{\{i \in \mathcal{M}: \kappa(i)=k\}} \frac{1}{z_i} \left(s_i^y\right)^{\frac{\sigma}{\sigma-1}} \left[ \left(1-\lambda_{\kappa(i)}\right)^{\frac{\sigma}{\sigma-1}} + \left(\lambda_{\kappa(i)}\right)^{\frac{\sigma}{\sigma-1}} \right] \right)^{-1}, \tag{23}$$

and so for a given amount of resources allocated to each market, how those resources get distributed across firms determines the labor productivity in each product market.

By contrast, the first-best level of labor productivity associated with an efficient allocation of resources satisfies an analogous expression to equation (23) with firm-level shares $s_i^y$ given by equation (22) in Proposition 2.1 , but with $\mu_i^n = \mu_i^y = 1$. If there is no market power dispersion, i.e. the ratio $\mu_i^n/\mu_i^y$ is constant across firms, then labor productivity is at its first-best level. Market power dispersion, whether it comes from heterogenous markups or markdowns, lowers labor productivity relative to the first-best because it induces an inefficient allocation of resources in product and labor markets: relative wages (prices) are not aligned with relative marginal revenue product of labor (marginal costs).[19]

**Trade, and Firm-level Market Power.** Trade affects the allocation of resources because it affects the amount of competition a firm faces in local and foreign markets. Higher levels of import competition reduces the share of the market that the domestic firm can capture, and because in this model market power is a function of the firm size, the markup the firm charges also falls. As Proposition 2.1 makes clear, the size of the firm in product and labor markets interact with each other through the endogenous markup and markdown of the firm, and so the higher level of import competition also affects the equilibrium share of the labor market that the firm captures. To compensate for the lower profitability in the product market the firm adjust in response her markdown on wages. Therefore, when markups are constant this effect is muted, and import competition cannot affect the allocation of resources within markets.[20]

Firm-level shares are also affected by the level of demand in the Foreign market. A higher demand in Foreign requires a larger scale of production by the firm. This can only be accomplished

[18]The amount of workers employed in product market $k$ is defined as:

$$N_k = \sum_{\{i \in \mathcal{K}: \kappa(i)=k\}} n_i.$$

Note that $N_k$ does not coincide with any of the market-level labor supply indexes in the model since those aggregate employment taking into account the amount of substitution across firms, possibly located in different labor markets.

[19]This dispersion in market powers comes from firms charging heterogeneous markups on marginal costs as well as heterogenous markdowns on wages. Therefore, the extreme cases where either $\sigma = \epsilon$, and therefore markups are constant across firms, or $\eta = \theta$, and so markdowns are constant across firms, do not achieve the first-best alone.

[20]When $\sigma = \epsilon$ the model is one with constant markups across firms, with $\mu_i^y = \frac{\sigma}{\sigma-1}$. Proposition 2.2 defines a triangular system where equation (24) determines firm's payroll shares independent of $s_i^y$, and equation (22) determines the sales shares given the equilibrium values of $s_i^n$. Both sales and payroll shares are independent of the expenditure share on imported goods $\lambda_k$.

by hiring more workers through either an increase in wages or a reduction in markdowns.[21] Changes in trade costs then, shift the allocation of labor across firms to satisfy the higher foreign demand. Proposition 2.2 makes this point clear for the case where $\sigma = \epsilon$.[22] When $\sigma = \epsilon$ the model is one with constant markups across firms and firm shares are not affected by import competition, and so the dependence of payroll shares on iceberg cost is related to the firm's access to export markets.[23]

**Proposition 2.2.** Suppose that there are no fixed cost of exporting and that $\sigma = \epsilon$, and so firms charge a constant markup $\mu_i^y = \frac{\sigma}{\sigma-1}$. Then, for given values of market-level sales and payroll shares, $\{S_k^y\}_{k\in\mathcal{K}}$ and $\{S_l^n\}_{l\in\mathcal{L}}$, the equilibrium values of the firm-level shares $\{s_i^n, s_i^y\}_{i\in\mathcal{M}}$ solve the following triangular system of equations:

$$s_i^n = \frac{\mu^n\left(s_i^n\right)^{\frac{\sigma(1+\eta)}{\eta+\sigma}} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left[1+\tau_{\kappa(i)}^{-\sigma}\right]^{\frac{1+\eta}{\eta+\sigma}} \left(S_{\kappa(i)}^y\right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\ell(j)=\ell(i)\}} \mu^n\left(s_j^n\right)^{\frac{\sigma(1+\eta)}{\eta+\sigma}} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left[1+\tau_{\kappa(i)}^{-\sigma}\right]^{\frac{1+\eta}{\eta+\sigma}} \left(S_{\kappa(i)}^y\right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}}, \tag{24}$$

$$s_i^y = \frac{\mu^n\left(s_i^n\right)^{\frac{(\sigma-1)\eta}{\eta+\sigma}} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(S_{\ell(i)}^n\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=\kappa(i)\}} \mu^n\left(s_j^n\right)^{\frac{(\sigma-1)\eta}{\eta+\sigma}} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(S_{\ell(j)}^n\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}. \tag{25}$$

where $\mu^n(s) = \left[\frac{1+\eta}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s\right]^{-1}$.

*Proof.* See Appendix A.3. □

There are cases, however, where labor markets are insulated from trade shocks. This is the case when firms in a labor market produce a unique type of product and so the shock do not have a differential effect between firms in a labor market.[24] Payroll-shares are independent of any trade related variable, and therefore international trade cannot correct any distortions coming from firm's labor market power.[25] We summarize this result in Corollary 2.1.

**Corollary 2.1.** When firms in a labor market produce a single type of product, i.e. $\kappa(j) = \kappa(i)$ for all $i, j \in \mathcal{M}$ with $\ell(i) = \ell(j) = l$, the joint distribution of $\{\mu_i^n, s_i^n, z_i\}$ in market $l$ is determined under market equilibrium. The distribution of markdowns is independent of trade related variables. Therefore, trade cannot affect the competitiveness of labor market $l$.

*Proof.* It follows from Proposition 2.2. □

**Market Competition, and Firm-level Market Power.** In the model, firms in product markets compete against firms that operate in different labor markets. At the same time, firms in a labor

[21] We note, however, that relative to the case with no markdowns the amount of workers the firms hires is lower. This is because relative to the no-markdown case where the marginal cost is constant, labor market power makes the marginal cost an increasing function of the total output produced.

[22] Here we don't impose Assumption 2.2 to included costly trade. Iceberg trade cost are bigger than one, but fixed cost of exporting still equal zero.

[23] Import competition in product markets is captured by $\lambda_k$. When $\sigma = \epsilon$ the system of equations determining the values of $s_i^n$ and $s_i^y$ are independent of $\lambda_k$

[24] We say that markets are overlap when firms in a labor market produce a unique type of product, and therefore compete against the same set of firm in both the product and labor market. We come back to this point in the quantitative section, Section 4.

[25] Note that in labor markets where firms produce only one type of product equilibrium payroll-shares can be solved in equilibrium independent of the economy aggregates. In this cases, firm-level payroll shares satisfy the block recursive property of Berger et al. (2022a) in an open economy context.

market produce different types of goods, and so in the model, product and labor markets do not overlap with each other. In this type of situations, the amount of competition a firm faces in the labor (product) market becomes an attribute of the firm when competing in product (labor) markets. In Appendix A.5 we show how the amount of competition in labor markets shape the allocation of resources in product markets, with firms located in less competitive labor markets capturing a larger share of the product market relative to their more productive competitors. This is because the lower level of competition allows the firm to charge higher markdowns and therefore produce at a lower cost despite having a smaller productivity level. Therefore, the partial overlap between markets creates an interesting effect where not necessarily the most productive firm captures the largest share. This distinguish our set-up from previous papers that have worked with the Atkeson and Burstein (2008) structure which predicts a correlation of one between productivity and market share.[26]

# 3 Data

We now describe the data we use. First we give a description of the Australian dataset. Because in the model, and therefore for the quantitative exercise, the definition of product and labor markets is crucial we then explain how we construct these markets. We highlight facts about the degree of interconnectedness between product and labor markets, what we call *market overlap*, and facts about producer concentration in this data that are crucial for the calibration and the quantitative implications of the model.

## 3.1 Dataset

We use the Business Longitudinal Analysis Data Environment (BLADE) a product of the Australian Bureau of Statistics (ABS). BLADE is an economic data tool that combines tax, trade, and intellectual property data sourced from surveys conducted by the ABS Business Register and the Australian Taxation Office (ATO).[27]

**Type of Activity Units.** BLADE collects data on all active businesses in Australia between 2001 and 2019 and reports it at two different levels of aggregation. The first level is a firm or enterprise group defined according to Australia's Corporation Law and is standard to most datasets containing micro-level data. Enterprise Group are at the level at which financial and balance sheet accounts are maintained and from which a consolidated financial position can be derived. The second level of aggregation is called *Type of Activity Unit* or TAU and are related to the concept of establishment defined by the System of National Accounts. The main difference between the two is that producing units in BLADE are not restricted to a single physical location, and therefore there are cases in which multiple establishments are aggregated into a single TAU id. Whether a TAU is one or multiple establishments depends on whether the enterprise group can report productive and employment activities via a minimum set of data items such as sales, wages and salaries, and expenditures. For most cases a TAU id correspond to a single establishment, and for that reason in the paper we use the word *establishments* to refer to BLADE *TAUs* id.

[26] In Section 3 we show that partially overlapping markets in Australia is predominant feature in the data.
[27] See Hansell and Rafi (2018) for a short overview of BLADE.

**Information collected by BLADE.** ATO requires all businesses registered for good and service taxes (GST) to submit a *Business Activity Statements* (BAS) where they report information on total sales, export sales, capital and non-capital purchases, and wages, salaries and other payments. We complement this information with the *Business Income Tax* (BIT) section of BLADE which provides data on wealth from balance sheet information, such as assets, liabilities, debts and the like. Data on firm-level employment is recovered from the *Pay As you Go* (PAYG) unit of BLADE which uses businesses income tax withholding to recover the total number of workers employed (head-count) and an estimate of the number of full-time equivalent employees. Finally, we use the *Business Location* unit and the *Indicative data items* sections of BLADE to recover information on business location and their industry classification. Both of these variables are crucial to define the boundaries of product and labor markets in Australia and to assign firms to each of these markets, which in turn determines the level of competition.

**Import Shares.** We supplement the data in BLADE with detailed import data from Australia's Department of Foreign Affairs and Trade (DFAT). DFAT reports sectoral imports at the 10-digit level of the Australian Customs Tariff classification. We use a correspondence index, provided by ABS, to translate those into the *Australian and New Zealand Standard Industry Classification* from 2006 (ANZSIC2006), which is the industry classification reported in BLADE. This match gives us dissagregated import penetration ratios for each product market category.

### 3.2 Product and Labor Markets.

**Defining Product and Labor Markets.** The amount of market concentration in the Australian data is crucial for our model's quantitative implications: They determine the amount of competition within each market, and therefore the distribution of resources across firms. In order to compute how much concentration there is in the data we not only need to define what constitutes a market, but also assign firms to each of them.[28]

Our definition of labor and product markets combines geographical information of Australia with information on the type of goods available in the country. The assignment of firms to markets uses instead the firm's location and industry classification contained in BLADE . To be more specific, we define a labor market as a region-sector pair. This definition is meant to capture two things. First, that certain skills are required in order to produce a particular type of good, and therefore workers cannot supply their labor services to any industry. Second, labor is not perfectly mobile and so even workers with the necessary skills cannot offer their services to firms located in distant places.[29]

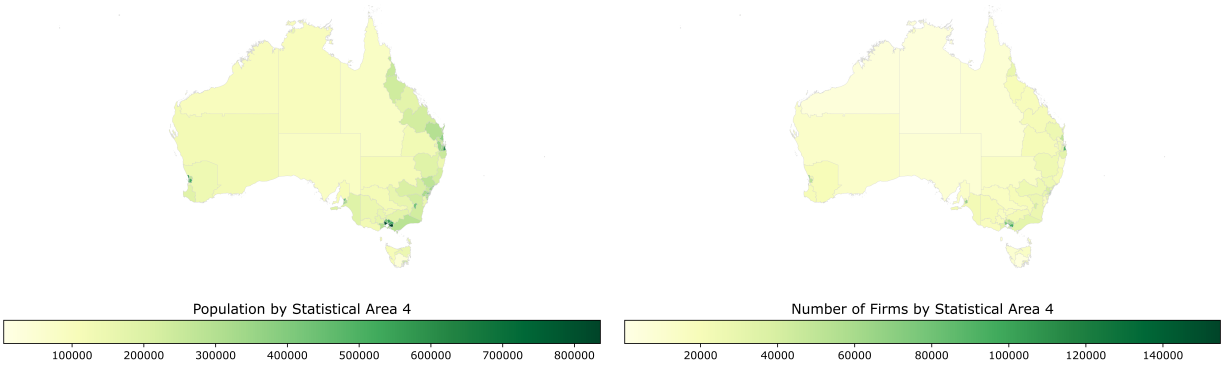
The choice of each component is not innocuous. We use a particular characteristic of our data to guide our choice. The Australian Statistical Geography Standard (ASGS) is a classification of Australia into a hierarchy of statistical areas, each designed to analyze a broad range of social, demographic, and economic statistics. The Statistical Area Level 4 (SA4s) classification are the largest sub-state regions in the ASGS and they are meant to represent labor markets or groups of labor markets within each state and territory.[30] They are designed to incorporate both where people live (labor supply) and where people work (labor demand), and therefore we use them for

[28] An important dimension of heterogeneity across firms in the model is the identity of the product and labor market where the firm operates

[29] For example, Manning and Petrongolo (2017) estimate that the attractiveness of jobs decays sharply with distance.

[30] SA4s are designed to represent the labor markets of the largest regional cities such as Wollongong, Bendigo and Townsville. In outer regional and remote areas, labor markets tend to be small and localized around regional towns. SA4s in these areas represent aggregations of these labor markets based on travel to work interactions, as well as industry and regional similarities.

**Figure 1:** SA4s by Population and Number of Firms




*Notes:* The figure plots the different Statistical Area Level 4 (SA4) of the Australian Statistical Geography Standard (ASGS) classification. SA4s are designed to represent the labor markets of the largest regional cities. The left-hand panel plots each SA4s by its population size, while the right-hand side plots SA4s by the total number of firms. Most of the economic activity, both in terms of population an number of firms, is concentrated around the eastern side of Australia where the cities of Brisbane, Sydney, and Melbourne are located.

the regional component in our definition of labor markets. There are 87 SA4s covering the whole of Australia without gaps or overlaps.[31] Figure 1 displays the division of Australia into SA4. Most SA4s have a population above 100,000 people. In cities, SA4s tend to have larger populations of 300,000 to 500,000 people (left-panel), and in this cities is where most of the firms are located (right-panel). The sectoral component in our definition is a *Group* of the Australia and New Zealand Standard Industry Classification from 2006 (ANZSIC06). There are 55 Groups in ANZSIC06 that correspond to a classification of industries at the 3-digit level.

Our definition of product markets does not depart from the literature. As in Edmond et al. (2015) and Gaubert and Itskhoki (2021) we base our definition of product markets on good's characteristics only. A product market is a 4-digit industry (or *Class*) of the ANZSIC06 classification.[32]

**Characteristics of Product and Labor Markets.** We restrict our sample to the manufacturing industry.[33] In total, we divide the Australian economy into 143 distinct product markets, and around 3569 labor markets.[34] The median labor market in Australia has around 12 firms competing with each other for workers, while the median product market has around 493 firms.

**Strong Market Interconnectedness.** We build a measure of market overlap to understand the degree of interconnectedness across markets in the Australia data. Our notion of market overlaps measures the extent to which the set of firms competing for workers in labor markets coincide, partially or completely, with the set of firms competing in product markets, the relevant notion to understand the propagation of trade shocks across seemingly unrelated product markets.

We say a labor market *overlaps* with a product market when all firms that operate in a labor

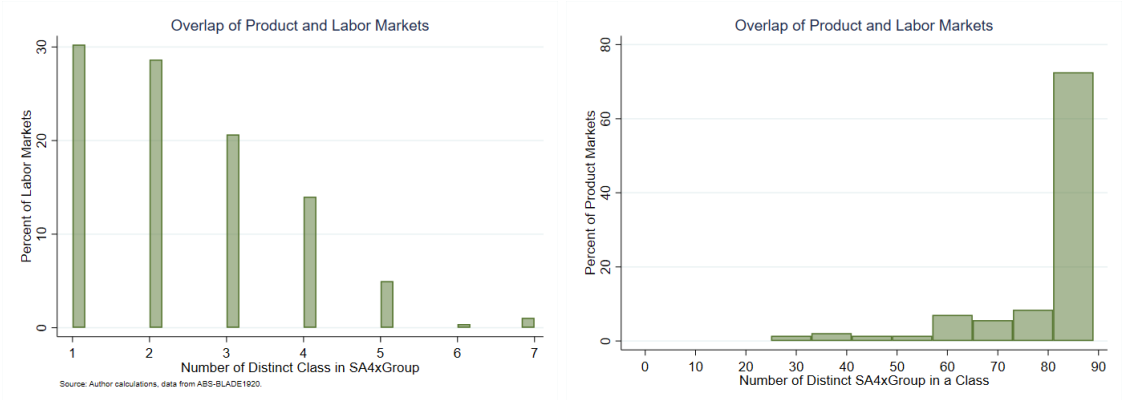
[31] Australian Statistical Geography Standard (ASGS) Edition 3.

[32] The 4-digit classification is the lowest level of aggregation at which we observed both the firm level data in BLADE and the import data in DFAT.

[33] Manufacturing businesses correspond to the Sub-Divisions 11 to 25 of ANZSCI06.

[34] There are 4785 possible SA4 times Group combination. In the data, only 3569 of those 4785 have a positive number of establishment.

**Figure 2:** Measuring Market Overlap in Australia - I




*Notes:* In the left-panel we plot the share of labor markets where the firms that operate in there belong to one, two, three and so forth, ***distinct product markets***. The right-panel plots the share of product markets where the firms that operates in there are located in only one, two and so forth labor markets. We say markets overlap when all firms that compete for workers (consumers) in a labor (product) market also compete for consumers (workers) in the product (labor) market.

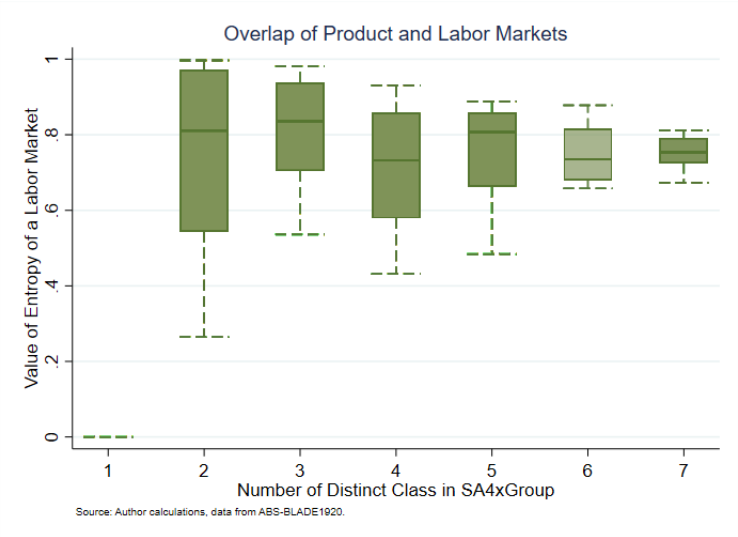
market are also part of the same product market. Instead, we say that a labor market partially overlaps with a product market when the firms in a labor market belong to multiple product markets. Note that when a labor market overlaps with a particular product market we can infer the product the firm produces by looking at which labor market this firms belongs to. We can similarly define a notion of market overlap based on product markets, and say that a product market overlaps with a labor market when all firms selling a particular product operate within a single labor market. When both product and labor markets overlap with each other we say there is *complete overlap* between both markets.

We measure the amount of overlap by counting the total number of *distinct product markets* that we observe in the data operate within a particular labor market. The left panel in Figure 2 plots the share of labor markets with one (i.e. market overlap), two, three, and so forth distinct product markets.The resulting distribution shows high levels of market overlap between labor and product markets in the Australian economy. Roughly 70% of labor markets provide workers to at most two product markets (30% to only one product market). Similarly, the right panel in Figure 2 plots the share of product markets that are produce within one (i.e. market overlap), two, three, and so forth distinct labor markets. The figure shows that the amount of market overlap is even stronger with more than 90% of product being produced in multiple labor markets.

A potential problem with this figure is that it does not take into account for example cases where multiple product markets hire labor from a single labor market, but a significant share of the market is captured by only one sector. In this case, we would ruled out market overlap when in fact there is. To overcome this issue we also compute a measure of entropy within each category where we previously identified partial overlap. The entropy is defined for the event: *firm* $i \in \mathcal{M}$ *belongs to product market* $k \in \mathcal{K}$, *given that* $i$ *belongs to labor market* $l \in \mathcal{L}$ *where there are* $x$ *distinct products being produced*; and it is given by,

$$H\left(K|\, L=l, X=x\right) = -\frac{1}{\log x}\sum_{k\in\mathcal{K}} p\left(K=k|\, L=l, X=x\right)\log p\left(K=k|\, L=l, X=x\right), \qquad (26)$$

**Figure 3:** Measuring Market Overlap in Australia - II




*Notes:* The figure shows, for a given count of distinct product markets $x$ the box plot of the distribution of the entropy $H\left(K|\, L=l, X=x\right)$ across labor markets $l$. Entropy is defined as in equation (26). The lower and upper value in each box plot represent the 10th and 90th percentile of the distribution instead of the usual minimum and maximum value because of data disclosing rules by ABS. The figure shows that cases where partial overlap was correspond with cases where in a labor market a non trivial share of firms produce different products.

where $H\left(\cdot\right)$ denotes the entropy, $p\left(\cdot\right)$ the probability of the event, $\mathcal{M}$ denotes the set of firms, and $\mathcal{K}$ and $\mathcal{L}$ denotes the set of product and labor markets, respectively.[35] Note that $H \in [0,1]$ and takes the value zero when there is no uncertainty about the realization of the event. On the other hand, the entropy equals one when the uncertainty is maximum, and this happens when the events $\{K=k|\, L=l, X=x\}$ has equal probability across the different values of $k$.

Based on Figure 2 the concern of miss measuring market overlap is greater for labor markets and for that reason we only show here the entropy for labor markets. Figure 3 plots for each $x$, the distribution of $H\left(K|\, L=l, X=x\right)$ across labor markets for a given count of distinct product markets $(x)$. The figure shows that our notion of market overlap is robust to this type of concern, and that market interconnectedness is strong in the data.

## 3.3 Concentration facts

The amount of concentration of sales and payrolls in product and labor markets is crucial for the model's quantitative implications. In this section we highlight facts about concentration for the manufacturing establishments in Australia.

**Concentration within Labor Markets** We measure a establishment's labor market share by its share of total payroll within a Statistical Area 4 times a 3-digit sector of ANZSIC06. The column label *Labor Market* in Table 1 shows that establishments within labor market are highly concentrated. The top 3 producers have a combined market share of of about 34 percent. The

[35] We estimate $p\left(\cdot\right)$ with the proportion of firms in labor market $l$ that produce goods in product market $k$.

**Table 1:** Concentration Facts in BLADE

| | Product Markets | Labor Markets |
|---|---|---|
| Within market concentration | | |
| Mean inverse HHI | 18.64 | 4.55 |
| Median inverse HHI | 4.43 | 2.52 |
| Mean cum. top 3 share | 0.021 | 0.338 |
| Median cum. top 3 share | 0.001 | 0.182 |
| Distribution of market shares | | |
| Mean share | 0.007 | 0.158 |
| Median share | 0.001 | 0.089 |
| p75 share | 0.005 | 0.221 |
| p95 share | 0.036 | 0.475 |
| SD share | 0.032 | 0.205 |
| Across markets concentration | | |
| p10 inverse HHI | 1.06 | 1.02 |
| p50 inverse HHI | 4.43 | 2.52 |
| p90 inverse HHI | 44.99 | 10.5 |
| p25 cum. top 3 share | 0.00001 | 0.048 |
| p50 cum. top 3 share | 0.001 | 0.182 |
| p75 cum. top 3 share | 0.004 | 0.598 |
| Market characteristics | | |
| Number of markets | 143 | 3569 |
| p25 number of establishments | 124 | 5 |
| p50 number of establishments | 493 | 12 |
| p75 number of establishments | 1786 | 31 |

*Notes:* The table shows descriptive statistcs on the amount of concentration in product and labor markets for the Australian manufacturing establishment, in 2018/2019 fiscal year. Observations in each labor market (product market) are weighted by the market's share of aggregate payrolls (aggregate sales).

median inverse Herfhindahl index (HHI) measure of concentration is around 2.5 and 4.6, which implies that the average labor market is as concentrated as they would be if the market were divided evenly across 5 establishments. It is also much lower than the 12 number establishments that we observe in the data.

The distribution of labor market shares is skewed to the right and extremely fat-tailed. The median market share of a establishment is 9 percent while the average market share is 15 percent. While the ninety-fifth percentile accounts only for 48 percent of total payrolls the ratio of the ninety-ninth percentile to the median is well above the threshold of 5.6.[36]

The overall patternthat emerges is one of very strong concentration. Although quite a few establishments operate in any given labor market, most of these establishments are small and a few large establishments account for most of the market's payroll.

**Concentration within Product Markets.** The column label *Product Market* in Table 1 repeats our previous analysis of market concentration but looking at product instead of labor markets. We measure a establishment's market share by its share of domestic sales revenue within a given 4-digit sector of ANZSIC06. The pattern that emerges from the data is one of less concentrated product

[36]ABS disclosure rules prevent us from reporting the value of the ninety-ninth percentile.

markets. The average 4-digit industry has a large number of establishments competing against each other, 493 establishments in product markets versus 12 establishments in labor markets. At the same time, the top 3 producers have a combined market share of about only 2 percent.

Despite this low level of concentration within product markets, we find that the median inverse Herfhindahl index (HHI) measure of concentration is about 4.43, much lower than the 493 producers that operates in a typical industry. This is a result of a few product markets that are highly concentrated that also account for a large share of aggregate sales.

The overall pattern is one of weak concentration in product markets. The difference between the levels of concentration in product and labor markets is in part explained by the less restrictive definition of product markets that do not consider any geographical restriction and therefore includes firms all around Australia.

# 4 Quantifying the model

To understand the role that international trade plays reducing distortions in product and labor markets we need to estimate our model. Table 2 summarizes the parameters in the model that need to be estimated to compute the pro-competitive gains from trade. On the production side we have five parameters. The two elasticity of substitution in the production technology of the final good: the elasticity of substitution across input varieties from a particular product market $\sigma$ and the elasticity of substitution across bundles of output from different product markets $\epsilon$. The other three parameters correspond to the two types of trade costs, the fixed costs of exporting $f_k$ and the iceberg cost $\tau_k$, and the firm-level technology $z_i$. There are also four parameters in the household's utility function. The disutility from labor $\bar{\varphi}$, the Frisch elasticity of labor supply $\varphi$, and the two elasticities of substitutions: across firms within a labor market $\theta$, and across labor markets $\eta$.

Additionally, we need to recover the labor supply and product demand shifters at the firm $\{\xi_{n,i}, \xi_{cy,i}\}$, and market level $\{\upsilon_{n,l}, \upsilon_{y,k}\}$, and the distribution of productivities for the Australian producers, $z_i$. Finally, we need to define the boundaries of each product and labor market, as well of the composition of firms operating in each of those markets.

Our strategy for estimation treats each firm in the data as a firm in the model, and therefore rationalizes the observed data as an equilibrium outcome of our theoretical environment. To recover the parameters we use the model to derive a set of equation that relates the desired parameters with a set of moments that we can measure in the data. Parameters are estimated by inverting this system. However, to invert this system we required knowledge of other primitives such as the market structure and the elasticities for which we rely on BLADE and estimates from the literature.[37] We use BLADE data, discussed in the previous section, to construct the set of moments for model. In particular we use information on firm-level sales, payroll, and wages, total imports by sector,and value of gross output.

The methodology we choose it does not come without costs. However, the advantages of this method outweights its constraints. In particular, it does not require to impose ad-hoc assumption on the fundamentals of the economy for either Australia or the rest of the world. Furthermore, since we treat the data as an equilibrium of the model we do not have to solve for the equilibrium in the open economy, which is not a trivial task, but only solve for the closed economy version of our model. The main issue with this approach is that it limits the type of questions that can be answered. For example we cannot consider counterfactual cases with more or less degrees

[37] In Section 5 we perform robustness to this choices.

**Table 2:** Parameters in the model

| | | | Source |
|---|---|---|---|
| Elasticities | Elasticity of subs. labor market | $\eta = 1.02, \theta = 0.8$ | Felix (2021) |
| | Elasticity of subs. product market | $\sigma = 2.5, \epsilon = 1.5$ | Sales to payroll ratio in BLADE |
| | Frisch elasticity labor supply | $\varphi = 0.25$ | Chetty (2012) |
| | Disutility of labor | $\bar{\varphi} = 1$ | Normalized |
| Market Structure | Number of firms | $\mathcal{M}$ | BLADE |
| | Number of prod. and labor mkt. | $\mathcal{K}$, $\mathcal{L}$ | BLADE |
| | Mapping firms to markets | $\kappa, \ell$ | BLADE |
| Shifters | Household pref. | $\xi_{n,i}$, and $\upsilon_{n,l}$ | Model Inversion |
| | Aggregate output. | $\xi_{y,i}$ and $\upsilon_{y,k}$ | Model Inversion |
| Technology | Productivity | $z_i$ | Model Inversion |
| | Trade costs | $\tau_k$ and $f_{x,i}$ | |

*Notes:* The table presents a list of the parameters required to compute the Pro-competitive gains from trade. The last column, labeled *Source*, specificies the type of information used to recover the parameter. BLADE indicates that it is recover directly from data using the Business Longitudinal Analysis Data Environment.

of openness to trade than the observed equilibrium, but instead look at limiting cases such as autarky.

## 4.1 Parametrization

**Elasticities.** We normalize the disutility of labor to $\bar{\varphi} = 1$ and we set the Frisch elasticity to $\varphi = 0.25$, a standard value in the literature (Chetty, 2012). For the four elasticities of substitutions our baseline results use estimates in the literature and information from BLADE, and then provide robustness to these choices.[38] We take the elasticity of substitution in labor markets from Felix (2021) which implies a within-market elasticity of $\eta = 1.02$ and an across-market elasticity of $\theta = 0.8$.

We chose the two elasticity in the product market so that the model matches the median value of the sales to payroll ratio in BLADE. The low levels of concentrations in sales shares implies that $\epsilon$ does not influence this ratio and therefore we set it to $\epsilon = 1.5$, a standard value in the literature (Atkeson and Burstein, 2008; Edmond et al., 2015; Amiti et al., 2019; Gaubert and Itskhoki, 2021; Deb et al., 2022). In the data, the median firm has sales that are three times larger than its total labor costs. For the model to match this moment, it requires a low value of $\sigma = 2.5$. Our value of $\sigma$ differs from those used in the previously cited papers, and therefore in Section 5 we perform robustness analysis to our choices of the parameters $\sigma$ and $\eta$.[39]

**Market Structure.** We treat each manufacturing establishment in BLADE as a firm in the model. We use the definition of product and labor markets outlined in Section 3.2. Therefore, we calibrate

[38] Appendix E we discuss how the 4 elasticities of substitution could be estimated using our firm-level data from Australia. However, this is still work in progress.

[39] We do not perform robustness analysis to $\epsilon$ and $\theta$ because there is less disagreement in the literature around these values. As we mentioned there is a long list of paper in the trade and macro literature that find an elasticity close to one for the upper nest in the CES goods aggregator. Similarly, Berger et al. (2022a), Felix (2021) and Deb et al. (2022) also finds that the elasticity in the upper nest is close to one for the CES aggregator for labor. In all these papers, the controversy is around the exact value of the elasticities in the inner nest.

the model to include around 30,000 manufacturing establishments operating in 143 different product markets and 3569 distinct local labor markets.

The dimension of the equilibrium is considerable given the granularity of the model which require us to solve for individual firm variables. However, in Appendix C.1.2, we show that solving for the equilibrium can be recast in terms of firm-level and market-level shares. This does not reduce the dimension of the problem, but constraints the search to a vector of numbers strictly between zero and one. Furthermore, once the equilibrium is written in terms of these shares, a nested fixed point algorithm can be implemented in which given market-level shares we can solved for the equilibrium values of the firm-level shares, similar to Proposition 2.1. Then, given the firm-level shares we can aggregate them up and solve for the market-level shares. These properties of the model makes solving for the equilibrium computationally tractable.

The nested fixed point algorithm, builds on the works of Berger et al. (2022a), with one substantial difference. In Berger et al. (2022a), the equilibrium shares can be solved market by market. Here, instead, we have to solved for all firm-level shares jointly because markets are partially overlapped.[40]

## 4.2 Model Inversion: Firm-level productivity and demand and supply shifters

The last set of parameters that we need to recover are the firm-level productivity, and the product demand and labor supply shifters. We show here how to recover those given values of the parameters $\{\eta, \theta, \sigma, \epsilon, \varphi, \bar{\varphi}\}$, and a partition of the economy into product and labor markets.

**Notation.** Moving forward it is important to define some notation for the set of observables used to invert the model. As in the model, we use $s_i^n$ to denote firm $i$'s payroll share, and $S_l^n$ for its market-level counterparts. In the same spirit, we use $w_i$ to denote the average wage paid by firm $i$. We use $d_i$ for firm $i$'s domestic sales and $s_{Hi}^y$ for the firm sales share. Finally, we use $M_k$ to denote the total expenditure in imported inputs of $k$-type products, and $S_k^y$ for the expenditure share in goods from product market $k$, this includes both domestic and imported goods.

**Model inversion.** Each of these observables have a counterpart in the model and are useful to pin-down the fundamentals for Australia. The information contained in firms' wages and their payroll shares allow us to recover the shifters in the labor supply equation for each firm,

$$\xi_{n,i} = \frac{s_i^n}{\left[\sum_{\{j \in \mathcal{M}: \ell(j)=\ell(i)\}} \left(\frac{w_i}{w_j}\right)^{1+\eta} s_j^n\right]}, \qquad \forall i \in \mathcal{M}, \tag{27}$$

and labor market,

$$\upsilon_{n,l} = \frac{S_l^n}{\left[\sum_{l' \in \mathcal{L}} \left(\frac{W_l}{W_{l'}}\right)^{1+\theta} S_{l'}^n\right]}, \qquad \forall l \in \mathcal{L}. \tag{28}$$

In the same way, information on firms' prices and their share in the domestic economy are

[40]The solution technique outlined in Appendix C.1.2, proved useful for solving even larger systems. The original version of this paper used to calibrate the model to the entire Australia economy with almost 700.000 firms, around 500 product markets, and close to 7000 labor markets.

informative of the product demand shifters for each firm,

$$\xi_{Hy,i} = \frac{s^y_{H,i}/p^{1-\sigma}_{H,i}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=\kappa(i)\}} \frac{s^y_{H,j}}{p^{1-\sigma}_{H,j}} + \sum_{\{j\in\mathcal{M}^*:\kappa(j)=\kappa(i)\}} \frac{s^y_{F,j}}{p^{1-\sigma}_{F,j}}}, \tag{29}$$

and product market,

$$\upsilon_{y,k} = \frac{S^y_k/P^{1-\epsilon}_k}{\sum_{r\in\mathcal{K}} \frac{S^y_r}{P^{1-\epsilon}_r}}, \qquad \forall k \in \mathcal{K}. \tag{30}$$

Finally, the information on firms' shares, firms' wages, and firm's prices inform us about the firm's productivity level,

$$z_i = \frac{\mu^y\left(s^y_{H,i}\right)}{\mu^n\left(s^n_i\right)} \frac{w_i}{p_{H,i}}, \qquad \forall i \in \mathcal{M}. \tag{31}$$

However, we lack information on firm prices $(p_{H,i}, p_{F,i})$, and on foreign firms' shares $(s^y_{F,j})$, and therefore we cannot identify all the parameters. Put it differently, there is more than one combination of firm's prices and demand shifters that are consistent with the same set of firm-level sales shares. One of those combinations has equilibrium prices that equal one for all sold goods in Australia, we labeled this economy the *Synthetic Australian Economy*.

**The Synthetic Economy (se).** The synthetic economy consists of a version of our original model except that they differs on the values of the firm-level technology parameters and on the values of the demand and supply shifters. These parameters are such that in the equilibrium of the synthetic economy all firm-level prices equal one. It follows that product market price indexes must also equal one,

$$P_k = \left[\sum_{i\in\mathcal{M}:\kappa(i)=k} \xi_{Hyi} + \sum_{i\in\mathcal{M}:\kappa(i)=k} \xi_{Fyi}\right]^{\frac{1}{1-\sigma}} = 1.$$

The second characteristic of our synthetic economy is that despite having unit prices, it still rationalizes the same set of observables as our original economy, and therefore it generates the same distribution of sales and payroll shares at the firm and market level.

Since the synthetic economy rationalizes the same data and in equilibrium prices equal one the parameters of the synthetic economy are uniquely determined by equations (27) to (31). We use upper script *se* to denote variables in our synthetic economy, and so we let $\left\{\xi^{se}_{n,i}, \xi^{se}_{Hy,i}, z^{se}_i\right\}_{\forall i\in\mathcal{M}}$,

$\left\{\upsilon^{se}_{y,k},\right\}_{k\in\mathcal{K}}$, and $\left\{\upsilon^{se}_{n,l}\right\}_{l\in\mathcal{L}}$ be the recovered parameters in the synthetic economy. They satisfy,[41]

$$\begin{aligned}
\xi^{se}_{n,i} &= \frac{s^n_i/w_i^{1+\eta}}{\sum_{\{j\in\mathcal{M}:\ell(j)=\ell(i)\}} s^n_j/w_j^{1+\eta}}, \qquad \forall i\in\mathcal{M},\\
\upsilon^{se}_{n,l} &= \frac{S^n_l/W_l^{1+\theta}}{\sum_{\{l'\in\mathcal{L}\}} S^n_{l'}/W_{l'}^{1+\theta}}, \quad \forall l\in\mathcal{L},\\
\xi^{se}_{Hy,i} &= s^y_{H,i}, \qquad \forall i\in\mathcal{M},\\
\upsilon^{se}_{y,k} &= S^y_k, \qquad \forall k\in\mathcal{K},\\
z^{se}_i &= \frac{\mu^y_i\left(s^y_{Hi}\right)}{\mu^n_i\left(s^n_i\right)} w_i, \qquad \forall i\in\mathcal{M}.
\end{aligned}$$

We note that, $\xi^{se}_{n,i}=\xi_{n,i}$ and $\upsilon^{se}_{n,l}=\upsilon_{n,l}$, because they are all constructed with the same data on firm wages and payroll shares.

Because all the relevant parameters for Australia are identified for the synthetic economy, it is possible to use the estimated parameters to compute the autarky equilibrium of this economy. The autarky equilibrium of the *se* economy is different from the autarky equilibrium we would like to compute. However, the two are related as stated in Proposition 4.1 where we used $\tilde{x}$ to denote variables in the autarky equilibrium. In particular, the two economies generate the same distribution of firm-level shares in autarky, and the same welfare levels.

**Proposition 4.1.** If $\left\{\tilde{w}^{se}_i,\tilde{p}^{se}_i,\tilde{s}^{n,se}_i,\tilde{s}^{y,se}_{H,i}\right\}_{i\in\mathcal{M}}$ is part of an autarky equilibrium in our artificial *se* economy, then, $\left\{\tilde{w}^{se}_i,\frac{z^{se}_i}{z_i}\tilde{p}^{se}_i,\tilde{s}^{n,se}_i,\tilde{s}^{y,se}_{H,i}\right\}_{i\in\mathcal{M}}$ is an equilibrium under autarky of the original economy. Furthermore, the synthetic economy and the true economy have the same welfare level in the autarky equilibrium.

*Proof.* See Appendix B.2. □

Furthermore, because the two economies rationalize the same initial equilibrium, the synthetic economy has the property that its autarky equilibrium predicts the same counterfactual changes in endogenous variables as the true economy, given the same proportional change in parameters. Therefore, although the two economies do not generate the same autarky equilibrium, this equilibrium is useful because it identifies counterfactual changes in the equilibrium of the original economy.[42] By solving the equilibrium of the synthetic economy we can study the welfare and efficiency gains, as well as the change in the distribution of market power, that result from closing Australia to international trade. We summarize this result in Proposition 4.2.

**Proposition 4.2.** Given the elasticity parameters $(\sigma,\epsilon,\eta,\theta,\varphi,\bar{\varphi})$, assignment functions $\kappa(\cdot)$ and $\ell(\cdot)$, and data on (i) firm level wages, (ii) firm-level payroll and domestic sales, and (iii) and expenditure shares on sectoral imports, the pro-competitive gains from closing the economy to international trade are identified.

*Proof.* See Appendix B.2. □

[41] The firm-level technology recovered in the synthetic economy is related to the true parameter $z_i$, by the equation,

$$z^{se}_i = p_{H,i} z_i,$$

and therefore measures the firm-level revenue productivity or TFPR (Hsieh and Klenow, 2009).

[42] A similar procedure for identification is used in Adão et al. (2022).

**Table 3:** Market Power Distribution: Trade versus Autarky

| | Markups | | Markdowns | | Market power ($\mu_i^y/\mu_i^n$) | |
|---|---|---|---|---|---|---|
| | Australia | Autarky | Australia | Autarky | Australia | Autarky |
| *Panel A. Firm-level moments* | | | | | | |
| Unconditional distribution | | | | | | |
| Mean | 1.669 | 1.6706 | 0.4974 | 0.4974 | 3.3583 | 3.3616 |
| Median | 1.6669 | 1.6671 | 0.5031 | 0.5031 | 3.3139 | 3.3143 |
| p75 | 1.6673 | 1.668 | 0.5044 | 0.5044 | 3.3474 | 3.3494 |
| p90 | 1.6689 | 1.671 | 0.5047 | 0.5047 | 3.4686 | 3.477 |
| p95 | 1.6719 | 1.6765 | 0.5048 | 0.5049 | 3.6366 | 3.6505 |
| Standard deviation (log) | 0.0111 | 0.0148 | 0.0286 | 0.0286 | 0.0329 | 0.0357 |
| $\log(p95/p50)$ | 0.002 | 0.003 | 0.001 | 0.002 | 0.040 | 0.042 |
| *Panel B. Market-level moments* | | | | | | |
| Across-market distribution | | | | | | |
| Mean | 1.7707 | 1.743 | 0.4878 | 0.4875 | | |
| Median | 1.6819 | 1.6891 | 0.4984 | 0.498 | | |
| p75 | 1.7171 | 1.754 | 0.8256 | 0.8226 | | |
| p90 | 1.8533 | 1.9067 | 0.5009 | 0.5007 | | |
| p95 | 2.8715 | 2.0511 | 0.5018 | 0.5017 | | |
| Standard deviation (log) | 0.1258 | 0.0634 | 0.0306 | 0.0304 | | |
| $\log(p95/p50)$ | 0.232 | 0.084 | 0.007 | 0.008 | | |
| *Panel C. Aggregate Implications* | | | | | | |
| Average import share | | | | | 0.3297 | 0 |
| Fraction of exporters | | | | | 0.091 | 0 |
| Gains from trade | | | | | – | -1.921 |
| Gains from trade, efficient allocation | | | | | – | -0.757 |

*Notes:* The table shows descriptive statistcs on the distribution of markups, markdowns, and total market power (the ratio of markups to markdowns) for the Australian manufacturing industry. The columns labeled *Australia* reports statistics for the observed trade equilibrium in the 2018/2019 fiscal year, while the columns labeled *Autarky* reports the same statistics at the counterfactual autarky equilibrium. Market-level markups are computed as the ratio of the quantity-weighted average marginal cost to the average price of the firms within a product market. Market-level markdowns are computed as the ratio of the employment-weighted average marginal revenue product of labor to the average wage of the firms within a labor market.

## 4.3 Market Power Distribution

Table 3 reports moments of the distribution of firms' markups and markdowns in our model. We compare these to an economy that is identical except that we shut down international trade.

**Initial Distribution.** Panel A of Table 3 reports moments of the unconditional distribution pooling over all firms across the different product and labor markets in Australia. The benchmark model implies an average markup of 1.669, a median markup of 1.6669, and a standard deviation of log

markups of 0.011. The low concentration in sales shares across firms in product markets implies that the distribution of markdowns has low dispersion and that most of the observations are centered slightly above the theoretical lower bound $\sigma/(\sigma-1) = 1.6667$. In fact, the ninety-fifth percentile of the markup distribution equals 1.672 almost half of the markup a pure monopolist would charge in the model, $\theta/(\theta-1) = 3$.

The distribution of markdowns imply by the model also features low levels of dispersion, the standard deviation of log markdowns is 0.029 slightly above the dispersion in markups. The model imposes bounds on the distribution of markdowns: A monopsonist charges the highest markdown which equals $\theta/(\theta+1) = 0.444$, while at the other extreme we have atomistic firms with markdowns equal to $\eta/(\eta+1) = 0.505$. Since in our calibration $\eta$ and $\theta$ are very close to each other the model predicts a low dispersion in markdowns despite the high levels of concentration in payroll shares within labor markets. The concentration in labor markets affects, however, where this distribution is centered. The average markdown equal to 0.4974, the median is 0.5031, and the ninety-fifth percentile is 0.5048 close to the monopsonist limit.

However, as we show in Section 2.4 what matters for the distribution of resources across firms is not the individual distribution of markups or markdowns, but the distribution of the ratio of the two. We refer to the ratio of a firm markup to its markdown as the total amount of market power the firm exerts, the last columns of Table 3 reports moments of this distribution. The model implies an average market power of 3.358, a median value of 3.314, and a standard deviation of log market power of 0.033. The distribution of market power exhibits more variation than the markups and the markdowns alone, and it is this dispersion what causes the misallocation of resources across firms.

We aggregate firm-level markups (markdowns) at the market-level using the sales (payroll) shares in the market as weights. Definition 4.1 makes explicit our aggregation procedure.[43] Panel B of Table 3 reports moments of the distribution of markups and markdowns across the different product and labor markets, respectively. Because large producers charge higher markups (markdowns), the average market-level markup (markdown) is 1.7707 (0.4878) much higher than the simple average. These market-level measures of market power are both larger and more disperse than their firm-level counterparts. The median labor market markdown is 0.4984 as opposed to 0.5031 while while the ninety-fifth percentile is 0.5018 as opposed to 0.5048. For product market markups the difference if even higher. The median product market markup is 1.6819 as opposed to 1.6669 while the ninety-fifth percentile is 2.8715 as opposed to 1.6719.

**Definition 4.1.** We let $\mu_l^n$ denote labor market $l$'s markdown, and $mu_k^y$ denote product market $k$'s markup across domestic producers. The quantities $\mu_l^n$ and $\mu_k^y$ are a weighted average of their firm-level counterparts using payroll and domestic sales as weights. Formally,

$$\frac{1}{\mu_l^n} = \sum_{\{i \in \mathcal{M}: \ell(i) = l\}} s_i^n \frac{1}{\mu_i^n}, \qquad \text{for all } l \in \mathcal{L},$$

and

$$\frac{1}{\mu_k^y} = \sum_{\{i \in \mathcal{M}: \kappa(i) = k\}} s_i^y \frac{1}{\mu_i^y}, \qquad \text{for all } K \in \mathcal{K}.$$

As shown in Felix (2021), under the nested-CES structure of labor supply, this aggregation procedures implies that our market-level markdown equals the ratio between the employment-

[43] We do not observe firm shares of foreign producers selling in Australia, and therefore we cannot use the model to back out the markups they charge to consumers. For this reason, our market-level markup only takes into account domestic producer. Since our counterfactual looks at the autarky equilibrium, where there are zero foreign sells, our measure of product market markup is still informative.

weighted average of a firm's wage and its marginal revenue product of labor. The nested-CES structure in product demand implies a similar result for the markup at the product market level, which in turn equals the ratio between the output-weighted average of a firm's price and its marginal cost of production. Furthermore, these market-level measures of market power are tightly linked with the amount of concentration in the market measured by its Herfindahl index. Proposition 4.3 summarize these results. Because our measured markup focus only on domestic producers the amount of concentration in the market is adjusted by the amount of import competition. The higher the level of concentration in a labor (product) market the larger it is the average markdown (markup) on wages (marginal cost). Thus, by exposing the economy more or less to international trade we affect the degree of market power in each market by inducing changes in the relative sizes of the firms.

**Proposition 4.3.** When labor supply is nested CES, and firms compete à la for Cournot for workers, the average wage markdown at labor market $l \in \mathcal{L}$ is given by:

$$\frac{1}{\mu_l^n} = \frac{\bar{r}_l}{\bar{w}_l} = 1 + \frac{1}{\theta} HHI_l + \frac{1}{\eta}\left(1 - HHI_l\right), \tag{32}$$

where $\bar{r}_l$ and $\bar{w}_l$ are market $l's$ employment-weighted average marginal revenue product of labor and wage, respectively, and $HHI_l = \sum_{\{i \in \mathcal{M}: \ell(i)=l\}} \left(s_i^n\right)^2$ the corresponding payroll Herfindahl index.

Similarly, when product demand is nested CES, and firms compete à la Cournot, the average markup charged by domestic producers at product market $k \in \mathcal{K}$ is given by:

$$\frac{1}{\mu_k^y} = \frac{\bar{mc}_k}{\bar{p}_{H,k}} = 1 - \frac{1}{\epsilon}\left(1 - \lambda_k\right) HHI_k^d - \frac{1}{\sigma}\left(1 - \left(1 - \lambda_k\right) HHI_k^d\right), \tag{33}$$

where $\bar{mc}_k$ and $\bar{p}_{Hk}$ are market $k's$ quantity-weighted average marginal cost and price, respectively, $HHI_k^d = \sum_{\{i \in \mathcal{M}: \kappa(i)=k\}} \left(s_i^y\right)^2$ is the corresponding domestic sales Herfindahl index, and $\lambda_k$ is the expenditure share on imported goods.

*Proof.* The first part of the statement it is proven in Felix (2021). To prove the second part of the statement a similar set of steps are required. For completeness we prove the proposition in Appendix D. □

**Distribution under autarky.** Consider now what happens when we shut down all international trade, which we report in Table 3, column labeled Autarky. The unconditional distribution markdowns remains almost invariant between the two equilibrium, while the distribution of markups is slightly shifted to the right. The average markdown is unchanged, the average markup increases from 1.669 to 1.6706, and there is a slight increase in the unconditional markup dispersion. Despite this, there is substantially more misallocation under autarky. As shown in panel C of Table 3, the benchmark economy implies welfare losses that are 2.5 higher than the welfare losses associated with the planning allocation. Hence the extent of misallocation is considerably worse under autarky.

As emphasized by Arkolakis et al. (2018), in several important theoretical benchmarks, the unconditional markup distribution is invariant to the level of trade costs. Therefore, if the changes in the moments of the unconditional markup distribution are used to infer the effects of trade on welfare and aggregate efficiency we can reach to wrong conclusions. While there is not an equivalent theoretical treatment for the relationship between trade and the distribution of markdowns, we show in Corollary 2.1 that there is a special case of our model where the same result holds true for

**Figure 4:** Firm Market Power: Autarky versus Trade

[Figure is being reviewed by ABS, and disclosure is pending.]

*Notes:* The figure plots the distribution of markups and markdowns for four different sets of firms in the Australian economy. The top-left panel plots the distribution of markups for the product market that before the shock was at the 95% of the distribution of import competition across the different product markets. We label this product market $k_{\lambda_{95}}$. The top-right panel plots the distribution of markdowns across all the firms that operate in product market $k_{\lambda_{95}}$. The bottom-left panel plots the markdown distribution in the labor market where the top producer in market $k_{\lambda_{95}}$, measured by its sales share, operates. Finally, the bottom-right panel plots the markdown distribution in the labor market where the majority of the firms in market $k_{\lambda_{95}}$ operate.

the unconditional distribution of firm-level markdowns. What matters is the joint distribution of market power and employment across producers.

We exemplify these forces by studying what happens to the distribution of market power and employment to a particular set of firms. Figure 4 plots the distribution of markups and markdowns for four different sets of firms in the Australian economy. The top-left panel plots the distribution of markups for the product market that before the shock was at the 95% of the distribution of import competition across the different product markets. For future reference we label this product market $k_{\lambda_{95}}$. The top-right panel plots the distribution of markdowns across all the firms that operate in product market $k_{\lambda_{95}}$. Because firms within a product market are heterogeneous in terms of the labor market from where they hire workers, we plot in the bottom-left panel the markdown distribution in the labor market where the top producer in market $k_{\lambda_{95}}$, measured by its sales share, operates. The bottom-right panel plots instead the markdown distribution in the labor market where the majority of the firms in market $k_{\lambda_{95}}$ operate.

In our benchmark model, closing the economy to trade dramatically shifts the markup distribution in product market $k_{\lambda_{95}}$. As these firms face lower competition given the absense of foreign competitors they gain market share and increase the markup charged to consumers in the product market. This gives these firms an advantage in their labor markets. As can be seen from the top-right panel, all of these firms also increase their markdowns on wages in their respective labor markets.

However, unlike product markets, where a significant share of the market is left for domestic producers to take, firms in labor markets are all operated by domestic producers. For this reason, after the shocks, not all shares can go up and so after closing the economy to trade there is a redistribution within the labor markets. While the most benefited firms manage to increase their size in labor markets, they do so at the expenses of their competitors. Closing the economy to international trade dramatically increases the markups of the largest producers where most employment is concentrated. As a result of this, misallocation increases resulting in larger welfare losses relative to the efficient allocation.

## 5 Gains From Trade

We focus on the gains due to a permanent reduction in trade costs and study the limit when these costs become prohibited.[44] We then ask our key question: to what extent does international trade reduce misallocation due to market power?

[44]This require us to solve for the autarky equilibrium. In Appendix C.1.2 we discuss the relevant equations and present an algorithm to solved for the equilibrium.

## 5.1 Total Gains from Trade

We measure the gains from trade by the percentage change in the household welfare from one equilibrium to another, which includes both the change in aggregate consumption as well as the change in the total number of hours worked. As reported in Table 4, for our baseline calibration closing the economy to international trade causes a 2% decrease in the household utility. This aggregate number, however, combines the effect of trade on product market power, labor market power, and on the interaction of the two. In order to decompose these forces, we quantify the welfare losses under different classes of models that are nested in our main specification, and report the results in Table 5.

We first look at the case when both product and labor markets are perfectly competitive, column *No Mkt Power*. In a perfectly competitive economy, the allocation of resources is efficient and there is no firm market power, and therefore sets the benchmark for our welfare numbers. The overall effect of trade is small in both the full and the perfectly competitive model, however, the welfare losses in the latter account for only 40% of the total effect. This highlights that, although the overall effects are small, they are entirely the result of the imperfectly competitive nature of product and labor markets. This numbers shows the importance of trade as a tool for correcting distortions caused by firm market power.

In columns *Markdowns* and *Markups* we works with a model where there are strategic complementarities in wage- and price-setting, respectively, but where we impose perfect competition in the other market. For example, the welfare losses reported under *Markups* are computed under a model where labor markets are perfectly competitive as in Atkeson and Burstein (2008) and Edmond et al. (2015). Both model with imperfect competition in only one market predict welfare losses from closing the economy to trade, and again these are considerably when compared to the efficient allocation. However, both of these models underestimate the welfare losses of closing the economy to trade, on average, by 55%. As it is standard in the literature, the lack of foreign competition increases the amount of market power domestic producers can exert in product markets distorting relative prices. Low-productivity firms grow larger than what they should be at the expenses of their high-productivity competitor. However, contrary to common belief, this is not followed by a decrease in firm's labor market power that could mitigate the previous effect. As we show in the previous section, the distribution of markdowns remains almost invariant as some firms expand while others contract after the shock. This reallocation follows the previous pattern with low-productivity firms growing too large which explains why we find negative welfare losses. Because product and labor market are inherently connected, these two forces amplify each other doubling the welfare losses in the full model.

Finally, in column *Constant MP*, we look at the case where $\eta = \theta$ and $\sigma = \epsilon$. In this limit, firms exert market power in both product and labor markets, but firm-level markups and markdowns are constant as can be seen from equation (18). We find that welfare losses are well approximated by models with constant levels of market power. Therefore, we find that the pro-competitive gains are elusive as defined by Arkolakis et al. (2018) when we also account for labor market power. This model, however, hides the mechanism through which the gains from trade materialize, a point to which we come back in the next section.

Table 5 also reports robustness to the gains from trade for different values of the elasticity of substitution within product and labor markets. In particular, we compute the welfare gains for different combinations of $\eta \in \{1.02, 10\}$ and $\sigma \in \{2.5, 5, 10\}$. These values correspond to combinations of the estimates in Felix (2021) and Berger et al. (2022a) for the elasticities in the labor market, and Edmond et al. (2015) and Gaubert and Itskhoki (2021) for the elasticities in the product market. We find that our results are not specific to the set of parameters chosen for the baseline specification.

**Table 4:** Welfare losses of closing the economy to international trade

| | No Mkt Power | Markups | Markdowns | Constant MP | Full model |
|---|---|---|---|---|---|
| Baseline $\eta = 1.02, \sigma = 2.5$ | 0.757 | 1.064 | 1.17 | 1.658 | 1.921 |
| $\eta = 1.02, \sigma = 5$ | 0.236 | 0.364 | 0.644 | 0.795 | 1.038 |
| $\eta = 1.02, \sigma = 10$ | 0.012 | 0.096 | 0.416 | 0.448 | 0.678 |
| $\eta = 10, \sigma = 5$ | 12.8 | 13 | 13.4 | 12.9 | 13.9 |
| $\eta = 10, \sigma = 10$ | 12.5 | 12.7 | 13.2 | 12.6 | 13.6 |

*Notes:* The table reports the welfare losses of moving Australia from the observed trade equilibrium in the 2018/2019 fiscal year to the counterfactual autarky equilibrium under different classes of models nested in our main specification labaled *Full model*. In *No Mkt Power* presents the both product and labor markets are perfectly competitive, and therefore represent the losses under the efficient allocation. In *Markdowns* and *Markups* there are strategic complementarities in wage- and price-setting, respectively, but we impose perfect competition in the other market. Finally, in column *Constant MP* firms exert market power in both product and labor markets, but firm-level markups and markdowns are constant.

The magnitude of the effects is sensitive to the value of these two parameter, however, they have the expected sign.

## 5.2 Pro-Competitive Gains from Trade

We finally look at the effectiveness of international trade to reduce market distortions, i.e., the amount of misallocation induced by markups and markdowns. We answer this question by studying the extent to which the losses from misallocation change as the economy is less exposed to trade.

We defined aggregate labor productivity in the model to be the quantity $A_{\mu^n,\mu^y}$ that in equilibrium satisfies the equation

$$Y = A_{\mu^n,\mu^y} \times N.$$

Aggregate productivity is an endogenous outcome of the model that depends on the distribution of the firm's productivities level, $z_i$, as well as in the allocation of resources across firms (see Section 2.4). Because firms exert market power in product and labor markets we usually have that $A_{\mu^n,\mu^y} \neq A_{\text{efficient}}$, where $A_{\text{efficient}}$ is the aggregate productivity attained under the efficient allocation of resources.

We measure the pro-competitive gains by looking at changes in allocative efficiency, defined as the change in aggregate labor productivity between the market equilibrium relative to the change in aggregate productivity under the efficient allocation. Formally, we defined the gains/losses in allocative efficiency as

$$\underbrace{\Delta \log A_{\mu^n,\mu^y}}_{\text{Total Change in TFP}} = \underbrace{\Delta \log A_{\text{efficient}}}_{\text{Productivity Gains}} + \underbrace{\Delta \log A_{\mu^n,\mu^y} - \Delta \log A_{\text{efficient}}}_{\text{Allocative efficiency gains}}.$$

The first term on the right-hand side of this expression gives the change in the first-best level of TFP, while the second term gives the change in misallocation, our measure of pro-competitive effects.

The pro-competitive gains will be positive (negative) if increased trade affects misallocation so that the change in aggregate productivity is larger (smaller) than the change in first-best productivity.

Table 5 reports the pro-competitive gains under the different classes of models discussed in the previous section. In the efficient allocation there is no misallocation of resources and pro-competitive gains are zero, as it is reflected in the column *No Mkt Power*. Furthermore, in a model with constant levels of market power ,by construction, there is no dispersion in firms' markups and markdowns. Resources are distributed across firms as in the efficient allocation, aggregate productivity equals first-best productivity, and there are also zero pro-competitive gains as reflected in the column *Constant MP*.[45]

**Table 5:** Allocative efficiency losses of closing the economy to international trade

| | No Mkt Power | Markups | Markdowns | Constant MP | Full model |
|---|---|---|---|---|---|
| Baseline $\eta = 1.02, \sigma = 2.5$ | 0 | 0.413 | 0.068 | 0 | 0.668 |
| $\eta = 1.02, \sigma = 5$ | 0 | 0.526 | 0.075 | 0 | 0.798 |
| $\eta = 1.02, \sigma = 10$ | 0 | 0.589 | 0.078 | 0 | 0.861 |
| $\eta = 10, \sigma = 5$ | 0 | 0.854 | 3.46 | 0 | 5.807 |
| $\eta = 10, \sigma = 10$ | 0 | 1.001 | 4.164 | 0 | 6.802 |

*Notes:* The table reports allocative efficiency losses of moving Australia from the observed trade equilibrium in 2018/2019 fiscal year to the counterfactual autarky equilibrium under different classes of models nested in our main specification labaled *Full model*. In *No Mkt Power* presents the both product and labor markets are perfectly competitive, and therefore represent the losses under the efficient allocation. In *Markdowns* and *Markups* there are strategic complementarities in wage- and price-setting, respectively, but we impose perfect competition in the other market. Finally, in column *Constant MP* firms exert market power in both product and labor markets, but there are no strategic complementarities between firm.

Pro-competitive losses from closing the economy to trade are 0.67%, as reported in Table 5. Our baseline estimate are small in magnitude, but the number is sensitive to the parameters of the model and they could be as high as 7 percent. As with our estimate of the welfare losses models that incorporate in their analysis market power in only one market underestimate substantially the total effect.

The allocative efficiency gains allows for a further decomposition to understand the role in product and labor market power driving the results. in particular, we decompose allocative efficiency gains into,

$$\Delta \log A_{\mu^n,\mu^y} - \Delta \log A_{\text{efficient}} = (\Delta \log A_{\mu^y} - \Delta \log A_{\text{efficient}}) + (\Delta \log A_{\mu^n,\mu^y} - \Delta \log A_{\mu^y}),$$

where the first term represents the allocative gains when only product market power is considered, and so the second term measures the contribution of labor market power and the interaction between the two.

Table 6 reports the contribution of labor market power to the allocative efficiency gains. While product market power accounts for the majority of the efficiency losses, labor market power is still

[45]The allocation of resources across firms is efficient in the sense that each firm captures the efficient share of each market. However, the total amount of resources produced in the economy is not efficient. The constant firm market power manifest as a wedge in the optimality condition between aggregate consumption and aggregate labor supply.

responsible for a third of the total effect . Furthermore, labor market power is not a negligible force in the model, since under possible parametrization it accounts for almost 90 percent of the total effect. In short, our results indicate that opening to trade is a useful tool to reduce market power induced misallocation.

**Table 6:** Contribution of labor market power to the allocative efficiency losses

| | Contribution Labor mkt power | Percentage of total effect |
|---|---|---|
| Baseline $\eta = 1.02, \sigma = 2.5$ | 0.255 | 38.2 |
| $\eta = 1.02, \sigma = 5$ | 0.272 | 34.1 |
| $\eta = 1.02, \sigma = 10$ | 0.272 | 31.6 |
| $\eta = 10, \sigma = 5$ | 4.953 | 85.3 |
| $\eta = 10, \sigma = 10$ | 5.801 | 85.3 |

*Notes:* The table reports the contribution of labor market power to the allocative efficiency losses from moving Australia from the observed trade levels in the 2018/2019 fiscal year to the counterfactual autarky equilibrium. Allocative efficiency is calculated as the change between these two equilibrium in aggregate labor productivity between the market equilibrium relative to the change under the efficient allocation.

# 6 Conclusion

We build a quantitative trade model with imperfect competition, strategic complementarities in price- and wage-setting, and a rich structure of product and labor markets to quantify the pro-competitive gains from international trade. The model is able to perform trade policy evaluations in a parsimonious way.

We combine our model with confidential firm-level data from Australia and study the gains/losses of moving Australia to autarky. The model matches exactly facts on import share dispersion, distribution of domestic market shares, and the cross-sectional relationship between import penetration and domestic concentration. The Australian data is characterized by a large amount of dispersion and concentration in firm payroll shares, and small concentration in sales shares which implies extensive missallocation through the lens of the model.

Using the model, we decompose the different channels by which the gains from trade materialize. We find that closing the economy to trade reduces welfare by 2% and increase resource misallocation by around 1% in our benchmark calibration. While the former are well approximated by models with constant levels of market power these models do not capture the losses in aggregate productive caused by misallocation. The size of these effects vary with the key parameters of the model and the structure of product and labor markets and our robustness exercise indicate that losses can go as high as 7%. In this sense, we find that accounting for imperfect competition in product and labor markets is quantitatively and economically meaningful.

From a policy viewpoint, our model suggests that obtaining large welfare gains from an improved allocation of resources may not require a detailed scheme of producer-specific subsidies and taxes that reduce the distortions associated with variable market power. Instead, simply opening an economy to trade may provide an excellent practical alternative that substantially improves productivity and welfare.

Looking forward, our results imply that the size of the pro-competitive gains depends on the values of four key parameters the two elasticity of substitution in product and in labor markets. Therefore, there is value in estimating those in Australia using our data. In Appendix E we took a first step on estimation, however, there is much to be done. Second, our results looked only at the aggregate effects of trade liberalization. Gains, however, are not ubiquitous, and therefore it is important to look at distributional effects as well.

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# Appendix

## A Proof and Derivations

### A.1 Cost function of the firm

In this section we derived the cost function of the firm. We show that the marginal cost of the firm is not constant as a function of firm level output despite the firm's linear technology. Hence, labor market power is a non-technological source of returns to scale.

The variable cost function of the firm is define as the solution to the problem:

$$c_i(y) \equiv \min_{n_i} w_i n_i,$$

subject to the technology of the firm $y = z_i n_i$ and the inverse labor supply equation (3) as the firm internalizes the market power she exerts over the labor input. Note that $y$ represents the total amount of output produced by the firm and does not distinguish whether it is sell in the domestic economy or abroad.

Because of the linear technology, given an amount of desired output $y$, optimal employment is just $n_i = \frac{y}{z_i}$. Plugging this into equation (3), and then into the objective we can compute the variable cost of the firm,

$$c(y) w_i \left(\frac{y}{z_i}\right) \frac{y}{z_i}. \tag{34}$$

The marginal cost of the firm implied by equation (34) is the product of two terms,

$$\frac{\partial c(y)}{\partial y} = \frac{w_i}{z_i} \times \left[\frac{\gamma_i^n + 1}{\gamma_i^n}\right].$$

As it is standard in models with perfectly competitive markets the marginal cost depend on the wage per efficiency units $w_i/z_i$. Differently from those models is the second term, related to the elasticity of labor supply $\gamma_i^n \equiv \left[\frac{\partial \log w_i}{\partial \log n_i}\right]^{-1}$, that appears because of the labor market power exerted by the firm. Since the elasticity of labor supply and the wages offer by the firm both depend on the scale of production $y$, the resulting marginal cost is not constant as a function of total output. Intuitively, if a firm decides to increase its scale of production it has to hire extra workers, but

because of the firm's labor market power this can only be done by offering a higher wage wage than its competitors. Since the firm has to offer a unique wage level to all its workers, this higher wage does not apply only to the marginal unit, but to all previously hired workers.

## A.2 Proof Proposition 2.1

**Definition Symmetric Countries.** To be concrete we start with a definition of what the symmetry countries assumptions implies.

**Definition A.1.** In a model with granular firms, two countries are say to be symmetric if one is a replica of the other..

Under Definition A.1, the Foreign country is partitioned into the same set of product and labor markets, with the same number of firms per market. Furthermore, each firm in Foreign has a counterpart in Home with the same level of productivity $z_i$.

**An expression for firm-level shares.** We start by noting that the share of domestic sales of firm $i$ can be written as:

$$s_{Hi}^{y} = \frac{p_{Hi}y_{Hi}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=\kappa(i)\}} p_{Hj}y_{Hj} + \sum_{\{j\in\mathcal{M}^*:\kappa(j)=\kappa(i)\}} p_{Fj}y_{Fj}} = s_i^{y}\left(1-\lambda_{\kappa(i)}\right),$$

where $\lambda_{\kappa(i)}$ is the import share in product market $\kappa(i)$, and $s_i^{y}$ is the share of domestic sales among domestic producers. Similarly, the sales share in the foreign country of firm $i$, $s_{H,i}^{y^*}$, can be written as:

$$s_{Hi}^{y^*} = \lambda_{\kappa(i)}^{*} s_i^{y^*},$$

where $\lambda_{\kappa(i)}^{*}$ is the import share of the Foreign country, and $s_i^{y^*}$ is the foreign sales shares of the domestic firms among exporters.

The three relevant shares for firm $i$ are then $\left\{s_i^n, s_i^y, s_i^{y*}\right\}_{i\in\mathcal{M}}$. These are given by:

$$
\begin{aligned}
s_i^n &= \frac{\xi_{n,i} w_i^{1+\eta}}{\sum_{\{j\in\mathcal{M}:\ell(j)=\ell(i)\}} \xi_{n,j} w_j^{1+\eta}}, \\
s_i^y &= \frac{\xi_{Hy,i} p_{Hi}^{1-\sigma}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=\kappa(i)\}} \xi_{Hy,j} p_{Hj}^{1-\sigma}}, \\
s_i^{y*} &= \frac{\xi_{Hy,i}^* p_{Hi}^{*1-\sigma}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=\kappa(i)\}} \xi_{Hy,j}^* p_{Hj}^{*1-\sigma}}.
\end{aligned}
\tag{35}
$$

We use the following equations: (i) the production technology of the intermediate good producers; (ii) firm $i \in \mathcal{M}$ labor supply and product demand equations ; and (iii) firm optimality summarize by the firm's the optimal pricing equation, to derive an expression for firm-level prices and wages. We start with the technology of the firm and use the labor supply equation to eliminate $n_i$, and the demand equation to eliminate quantities$y_{Hi}$ and $y_{Hi}^*$:

$$
\xi_{Hy,i}\left(\frac{p_{Hi}}{P_{\kappa(i)}}\right)^{-\sigma} Y_{\kappa(i)} + \xi_{Hy,i}^*\left(\frac{p_{Hi}^*}{P_{\kappa(i)}^*}\right)^{-\sigma} Y_{\kappa(i)}^* = z_i \xi_{n,i}\left(\frac{w_i}{W_{\ell(i)}}\right)^{\eta} N_{\ell(i)}.
$$

We then use the optimal pricing equation of the firm $p_{H,i} = \frac{\mu_i^y}{\mu_i^n}\frac{w_i}{z_i}$ and $p_{Hi}^* = \frac{\mu_i^y}{\mu_i^n}\tau_{\kappa(i)}\frac{w_i}{z_i}$ to express everything in term of firm-level wages. We then solve for wages to get the equation:

$$
w_i = z_i^{\frac{\sigma-1}{\eta+\sigma}}\left[\frac{\xi_{Hy,i}}{\xi_{n,i}}\left(\frac{\mu_i^y}{\mu_i^n}\right)^{-\sigma}\frac{P_{\kappa(i)}Y_{\kappa(i)}/P_{\kappa(i)}^{1-\sigma}}{W_{\ell(i)}N_{\ell(i)}/W_{\ell(i)}^{1+\eta}} + \phi_i\frac{\xi_{Hy,i}^*}{\xi_{n,i}}\left(\frac{\mu_i^{y^*}}{\mu_i^n}\tau_{\kappa(i)}\right)^{-\sigma}\frac{P_{\kappa(i)}^*Y_{\kappa(i)}^*/P_{\kappa(i)}^{*1-\sigma}}{W_{\ell(i)}N_{\ell(i)}/W_{\ell(i)}^{1+\eta}}\right]^{\frac{1}{\eta+\sigma}}, \tag{36}
$$

which implies prices are:

$$
\begin{aligned}
p_{Hi} &= \frac{\mu_i^y}{\mu_i^n} z_i^{-\frac{1+\eta}{\eta+\sigma}}\left[\frac{\xi_{Hy,i}}{\xi_{n,i}}\left(\frac{\mu_i^y}{\mu_i^n}\right)^{-\sigma}\frac{P_{\kappa(i)}Y_{\kappa(i)}/P_{\kappa(i)}^{1-\sigma}}{W_{\ell(i)}N_{\ell(i)}/W_{\ell(i)}^{1+\eta}} + \phi_i\frac{\xi_{Hy,i}^*}{\xi_{n,i}}\left(\frac{\mu_i^{y^*}}{\mu_i^n}\tau_{\kappa(i)}\right)^{-\sigma}\frac{P_{\kappa(i)}^*Y_{\kappa(i)}^*/P_{\kappa(i)}^{*1-\sigma}}{W_{\ell(i)}N_{\ell(i)}/W_{\ell(i)}^{1+\eta}}\right]^{\frac{1}{\eta+\sigma}}, \\
p_{Hi}^* &= \tau_{\kappa(i)}\frac{\mu_i^{y^*}}{\mu_i^y} p_{Hi}.
\end{aligned}
\tag{37}
$$

We can plug equation (36) into equation (35) to derive our expressions for the firm's share.

We start computing the firm-level payroll shares. We first use equations (35) and (36) to write

$s_i^n$ as:

$$s_i^n = \frac{\xi_{n,i} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left[ \frac{\xi_{Hy,i}}{\xi_{n,i}} \left( \frac{\mu_i^y}{\mu_i^n} \right)^{-\sigma} \frac{P_{\kappa(i)} Y_{\kappa(i)}}{P_{\kappa(i)}^{1-\sigma}} + \phi_i \frac{\xi^*_{Hy,i}}{\xi_{n,i}} \left( \frac{\mu_i^{y^*}}{\mu_i^n} \tau_{\kappa(i)} \right)^{-\sigma} \frac{P^*_{\kappa(i)} Y^*_{\kappa(i)}}{P^{*1-\sigma}_{\kappa(i)}} \right]^{\frac{1+\eta}{\eta+\sigma}}}{\sum_{\{j \in \mathcal{M}: \ell(j)=\ell(i)\}} \xi_{n,j} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left[ \frac{\xi_{Hy,j}}{\xi_{n,j}} \left( \frac{\mu_j^y}{\mu_j^n} \right)^{-\sigma} \frac{P_{\kappa(j)} Y_{\kappa(j)}}{P_{\kappa(j)}^{1-\sigma}} + \phi_j \frac{\xi^*_{Hy,j}}{\xi_{n,j}} \left( \frac{\mu_j^{y^*}}{\mu_j^n} \tau_{\kappa(j)} \right)^{-\sigma} \frac{P^*_{\kappa(j)} Y^*_{\kappa(j)}}{P^{*1-\sigma}_{\kappa(j)}} \right]^{\frac{1+\eta}{\eta+\sigma}}}. \quad (38)$$

Note that similar Berger et al. (2022a) labor market aggregates cancel out since those affect all the firms in the market alike. Different from Berger et al. (2022a) firm-level shares still depend on market level aggregates related to product markets. This is because firms in a labor market may or not participate in foreign trade and therefore the scale of production of some firms are affected by the level of demand in the Foreign economy. Second, firms in a labor market produce different types of products within the Home economy and therefore are differentially exposed to the demand conditions across these different product markets.

The next step is to express the market-level aggregates in terms of market-level shares. To do that we note that in the model the following relationship holds true for all product markets:

$$Y_k = \upsilon_{yk} \left( \frac{P_k}{P} \right)^{-\epsilon} Y, \quad \text{for all } k \in \mathcal{K},$$

and therefore, product market $k$ sales share of total revenue, defined as $S_k^y \equiv \frac{P_k Y_k}{PY}$, in equilibrium equals

$$S_k^y = \upsilon_{y,k} \left( \frac{P_k}{P} \right)^{1-\epsilon}.$$

Putting these two equations together, and using the symmetric countries assumption, we get,

$$\frac{P_{\kappa(i)} Y_{\kappa(i)}}{P_{\kappa(i)}^{1-\sigma}} = \frac{P^*_{\kappa(i)} Y^*_{\kappa(i)}}{\left( P^*_{\kappa(i)} \right)^{1-\sigma}} = \upsilon_{y,\kappa(i)} \left( \frac{S^y_{\kappa(i)}}{\upsilon_{y,\kappa(i)}} \right)^{\frac{\sigma-\epsilon}{1-\epsilon}} \frac{PY}{P^{1-\sigma}}. \quad (39)$$

Combining equations (38) with equation (39) and using again the symmetric countries property

payroll-shares are:

$$s_i^n = \frac{\xi_{n,i} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left[ \frac{\xi_{Hy,i}}{\xi_{n,i}} \left( \frac{\mu_i^n}{\mu_i^y} \right)^\sigma + \phi_i \frac{\xi^*_{Hy,i}}{\xi_{n,i}} \left( \frac{\mu_i^n}{\mu_i^{y^*}} \right)^\sigma \tau_{\kappa(i)}^{-\sigma} \right]^{\frac{1+\eta}{\eta+\sigma}} \upsilon_{y,\kappa(i)}^{\frac{(1+\eta)(\sigma-1)}{(\eta+\sigma)(\epsilon-1)}} \left( S^y_{\kappa(i)} \right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}}{\sum_{\{j \in \mathcal{M}: \ell(j)=\ell(i)\}} \xi_{n,j} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left[ \frac{\xi_{Hy,j}}{\xi_{n,j}} \left( \frac{\mu_j^n}{\mu_j^y} \right)^\sigma + \phi_j \frac{\xi^*_{Hy,j}}{\xi_{n,j}} \left( \frac{\mu_j^n}{\mu_j^{y^*}} \right)^\sigma \tau_{\kappa(j)}^{-\sigma} \right]^{\frac{1+\eta}{\eta+\sigma}} \upsilon_{y,\kappa(j)}^{\frac{(1+\eta)(\sigma-1)}{(\eta+\sigma)(\epsilon-1)}} \left( S^y_{\kappa(j)} \right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}}. \tag{40}$$

To compute the domestic sales share we first note use equation (35) and (37) to substitute prices out of the definition of $s_i^y$, and use equation (39) to deal with product market level aggregates. For labor market level aggregates we note that they satisfy an analogous form to equation (39):

$$\frac{W_{\ell(j)} N_{\ell(j)}}{W_{\ell(j)}^{1+\eta}} = \upsilon_{n,\ell(j)} \left( \frac{S^n_{\ell(j)}}{\upsilon_{n,\ell(j)}} \right)^{\frac{\theta-\eta}{1+\theta}} \frac{WN}{W^{1+\eta}}. \tag{41}$$

Domestic sales share are then,

$$s_i^y = \frac{\xi_{Hy,i} \left( \frac{\mu_i^n}{\mu_i^y} \right)^{\sigma-1} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left[ \frac{\xi_{Hy,i}}{\xi_{n,i}} \left( \frac{\mu_i^n}{\mu_i^y} \right)^\sigma + \phi_i \frac{\xi^*_{Hy,i}}{\xi_{n,i}} \left( \frac{\mu_i^n}{\mu_i^{y^*}} \right)^\sigma \tau_{\kappa(i)}^{-\sigma} \right]^{\frac{1-\sigma}{\eta+\sigma}} \upsilon_{n,\ell(i)}^{\frac{(1+\eta)(\sigma-1)}{(\eta+\sigma)(1+\theta)}} \left( S^n_{\ell(i)} \right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}{\sum_{\{j \in \mathcal{M}: \kappa(j)=\kappa(i)\}} \xi_{Hy,j} \left( \frac{\mu_j^n}{\mu_j^y} \right)^{\sigma-1} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left[ \frac{\xi_{Hy,j}}{\xi_{n,j}} \left( \frac{\mu_j^n}{\mu_j^y} \right)^\sigma + \phi_j \frac{\xi^*_{Hy,j}}{\xi_{n,j}} \left( \frac{\mu_j^n}{\mu_j^{y^*}} \right)^\sigma \tau_{\kappa(j)}^{-\sigma} \right]^{\frac{1-\sigma}{\eta+\sigma}} \upsilon_{n,\ell(j)}^{\frac{(1+\eta)(\sigma-1)}{(\eta+\sigma)(1+\theta)}} \left( S^n_{\ell(j)} \right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}, \tag{42}$$

$$s_i^{y*} = \frac{\phi_i \xi^*_{Hy,i} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left( \frac{\mu_i^n}{\mu_i^{y^*}} \right)^{(\sigma-1)} \left[ \frac{\xi_{Hy,i}}{\xi_{n,i}} \left( \frac{\mu_i^n}{\mu_i^y} \right)^\sigma + \phi_i \frac{\xi^*_{Hy,i}}{\xi_{n,i}} \left( \frac{\mu_i^n}{\mu_i^{y^*}} \right)^\sigma \tau_{\kappa(i)}^{-\sigma} \right]^{\frac{1-\sigma}{\eta+\sigma}} \upsilon_{n,\ell(j)}^{\frac{(1+\eta)(\sigma-1)}{(\eta+\sigma)(1+\theta)}} \left( S^n_{\ell(j)} \right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}{\sum_{\{j \in \mathcal{M}: \kappa(j)=\kappa(i)\}} \phi_j \xi^*_{Hy,j} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left( \frac{\mu_j^n}{\mu_j^{y^*}} \right)^{(\sigma-1)} \left[ \frac{\xi_{Hy,j}}{\xi_{n,j}} \left( \frac{\mu_j^n}{\mu_j^y} \right)^\sigma + \phi_j \frac{\xi^*_{Hy,j}}{\xi_{n,j}} \left( \frac{\mu_j^n}{\mu_j^{y^*}} \right)^\sigma \tau_{\kappa(j)}^{-\sigma} \right]^{\frac{1-\sigma}{\eta+\sigma}} \upsilon_{n,\ell(j)}^{\frac{(1+\eta)(\sigma-1)}{(\eta+\sigma)(1+\theta)}} \left( S^n_{\ell(j)} \right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}. \tag{43}$$

Now we use Assumption 2.1 (all demand and supply shifters equal one) and Assumption 2.2 (trade is free between the two countries)in the main text. Under these assumptions we have that payroll shares are,

$$s_i^n = \frac{z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left[ \left( \frac{\mu_i^n}{\mu_i^y} \right)^\sigma + \left( \frac{\mu_i^n}{\mu_i^{y^*}} \right)^\sigma \right]^{\frac{1+\eta}{\eta+\sigma}} \left( S^y_{\kappa(i)} \right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}}{\sum_{\{j \in \mathcal{M}: \ell(j)=\ell(i)\}} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left[ \left( \frac{\mu_j^n}{\mu_j^y} \right)^\sigma + \left( \frac{\mu_j^n}{\mu_j^{y^*}} \right)^\sigma \right]^{\frac{1+\eta}{\eta+\sigma}} \left( S^y_{\kappa(j)} \right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}}, \tag{44}$$

domestic sales share are $s^y_{Hi} = s^y_i \left(1 - \lambda_{\kappa(i)}\right)$ with,

$$s^y_i = \frac{\left(\frac{\mu^n_i}{\mu^y_i}\right)^{\sigma-1} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left[\left(\frac{\mu^n_i}{\mu^y_i}\right)^{\sigma} + \left(\frac{\mu^n_i}{\mu^{y^*}_i}\right)^{\sigma}\right]^{\frac{1-\sigma}{\eta+\sigma}} \left(S^n_{\ell(i)}\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=\kappa(i)\}} \left(\frac{\mu^n_j}{\mu^y_j}\right)^{\sigma-1} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left[\left(\frac{\mu^n_j}{\mu^y_j}\right)^{\sigma} + \left(\frac{\mu^n_j}{\mu^{y^*}_j}\right)^{\sigma}\right]^{\frac{1-\sigma}{\eta+\sigma}} \left(S^n_{\ell(j)}\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}, \tag{45}$$

and sales share in Foreign are $s^{y^*}_{Hi} = s^{y^*}_i \lambda^*_{\kappa(i)}$ with,

$$s^{y*}_i = \frac{z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(\frac{\mu^n_i}{\mu^{y^*}_i}\right)^{(\sigma-1)} \left[\left(\frac{\mu^n_i}{\mu^y_i}\right)^{\sigma} + \left(\frac{\mu^n_i}{\mu^{y^*}_i}\right)^{\sigma}\right]^{\frac{1-\sigma}{\eta+\sigma}} \left(S^n_{\ell(i)}\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=\kappa(i)\}} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(\frac{\mu^n_j}{\mu^{y^*}_j}\right)^{(\sigma-1)} \left[\left(\frac{\mu^n_j}{\mu^y_j}\right)^{\sigma} + \left(\frac{\mu^n_j}{\mu^{y^*}_j}\right)^{\sigma}\right]^{\frac{1-\sigma}{\eta+\sigma}} \left(S^n_{\ell(j)}\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}. \tag{46}$$

Furthermore, symmetry between countries with free trade implies that in equilibrium $(1 - \lambda_k)$ must equal $\lambda^*_k$ for all $k \in \mathcal{K}$, and therefore from equations (45) and (46) that $s^y_i = s^{y^*}_i$ is a solution of the system, for any set of values of firm payroll shares $\{s^n_i\}_{i\in\mathcal{M}}$, and market-level shares. Replacing this back into equation (44) to (46) we get the result in Proposition 2.1:

$$s^n_i = \frac{\left(\frac{\mu^n(s^n_i)}{\mu^y\left(s^y_i\left(1-\lambda_{\kappa(i)}\right)\right)}\right)^{\frac{(1+\eta)\sigma}{\eta+\sigma}} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(S^y_{\kappa(i)}\right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\ell(j)=\ell(i)\}} \left(\frac{\mu^n\left(s^n_j\right)}{\mu^y\left(s^y_j\left(1-\lambda_{\kappa(j)}\right)\right)}\right)^{\frac{(1+\eta)\sigma}{\eta+\sigma}} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(S^y_{\kappa(j)}\right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}},$$

$$s^y_i = \frac{\left(\frac{\mu^n(s^n_i)}{\mu^y\left(s^y_i\left(1-\lambda_{\kappa(i)}\right)\right)}\right)^{\frac{(\sigma-1)\eta}{\eta+\sigma}} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(S^n_{\ell(i)}\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=\kappa(i)\}} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(\frac{\mu^n\left(s^n_j\right)}{\mu^y\left(s^y_j\left(1-\lambda_{\kappa(i)}\right)\right)}\right)^{\frac{(\sigma-1)\eta}{\eta+\sigma}} \left(S^n_{\ell(j)}\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}$$

with,

$$\mu^n_i = \left[\frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s^n_i\right]^{-1}, \quad \text{and} \quad \mu^y_i = \left[\frac{\sigma-1}{\sigma} - \left(\frac{1}{\epsilon} - \frac{1}{\sigma}\right) s^y_i \left(1 - \lambda_{\kappa(i)}\right)\right]^{-1}.$$

## A.3 Proof Proposition 2.1

Here we don't impose Assumption 2.2.[46] When $\sigma = \epsilon$, the markup charged by the firm in both Home and Foreign is constant and equals $\mu^y = \frac{\sigma}{\sigma-1}$. Then from equation (40), we have that payroll-shares

[46] We keep the iceberg cost but impose no fixed cost of exporting.

are given by:

$$s_i^n = \frac{\left(\mu_i^n\right)^{\frac{\sigma(\eta+1)}{\eta+\sigma}} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(1+\tau_{\kappa(i)}^{-\sigma}\right)^{\frac{1+\eta}{\eta+\sigma}} \left(S_{\kappa(i)}^y\right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\ell(j)=\ell(i)\}} \left(\mu_i^n\right)^{\frac{\sigma(\eta+1)}{\eta+\sigma}} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(1+\tau_{\kappa(j)}^{-\sigma}\right)^{\frac{1+\eta}{\eta+\sigma}} \left(S_{\kappa(j)}^y\right)^{\frac{\sigma-\epsilon}{1-\epsilon}\frac{1+\eta}{\eta+\sigma}}}. \tag{47}$$

A reduction in trade costs changes the allocation of labor, however this reallocation of labor across firms does not come from an increase in import competition since markups are constants, and as it is clear from equation (47) neither $\lambda_k$ or $s_i^y$ determines the level of $s_i^n$. The reallocation of labor across firms comes instead from the lower costs that the firm face to export its goods the Foreign country. Because of the smaller iceberg costs firm requires more labor to satisfy the Foreign demand. We note, however, that relative to a case with no markdowns the amount of labor that firms hires is lower because to satisfy the demand implies increasing total cost due to labor market power. From equations 43, sales shares are:

$$s_i^{y*} = s_i^y = \frac{\left(\mu_i^n\right)^{\frac{(\sigma-1)\eta}{\eta+\sigma}} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(S_{\ell(i)}^n\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=\kappa(i)\}} \left(\mu_j^n\right)^{\frac{(\sigma-1)\eta}{\eta+\sigma}} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}} \left(S_{\ell(j)}^n\right)^{\frac{\theta-\eta}{1+\theta}\frac{\sigma-1}{\eta+\sigma}}}. \tag{48}$$

## A.4 Proof Corollary 2.1

To show the Corollary, notice that when all firms produce for the same product market the term in the numerator $\left(\left(1+\tau_{\kappa(i)}^{-\sigma}\right) S_{\kappa(i)}^{y\frac{\sigma-\epsilon}{1-\epsilon}}\right)^{\frac{1+\eta}{\eta+\sigma}}$ cancels out and we get,

$$s_i^n = \frac{\left(\mu_i^n\right)^{\frac{\sigma(\eta+1)}{\eta+\sigma}} z_i^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}}}{\sum_{\{j\in\mathcal{M}:\ell(j)=\ell(i)\}} \left(\mu_i^n\right)^{\frac{\sigma(\eta+1)}{\eta+\sigma}} z_j^{\frac{(1+\eta)(\sigma-1)}{\eta+\sigma}}}.$$

which is independent of any trade related variable, and therefore so it is $s_i^y$.

## A.5 Market Competition and Firm Market Power

We focus on three particular markets, two labor markets $l_1$ and $l_2$, and one product market $k$. These three markets are interconnected because all the firms in labor market $l_1$ and $l_2$ produce for product market $k$. Furthermore, they are the only producers of market $k$ goods, and therefore there is no foreign competition nor competition from other domestic firms outside labor markets $l_1$ and $l_2$.

Firms in labor market $l_1$ differ from firms in labor market $l_2$ in two dimensions. First, the productivity of the representative firm in labor market $l_1$, $z_1$, differs from the productivity level of the representative firm in labor market $l_2$, $z_2$. They also differ in the total number of firms operating in each market $M_1$ and $M_2$, respectively. This implies that in product market $k$, there are two type of firms : There are $M_1$ firms with productivity $z_1$ and $M_2$ firms with productivity $z_2$, all competing against each other in product market $k$.

The allocation of resources across these three markets is described by the equations:

- The labor supply equation:

$$w_i = \bar{\omega}_n \left(n_i\right)^{\frac{1}{\eta}} N_{\ell(i)}^{\frac{1}{\theta}-\frac{1}{\eta}}, \quad N_{\ell(i)} = \left[\sum_{\{j:\ell(j)=\ell(i)\}} \left(n_j\right)^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{1+\eta}}, \tag{49}$$

  where $\bar{\omega}_n \equiv W N^{-\frac{1}{\theta}}$.

- The product demand equation:

$$p_i = \bar{\omega}_y y_i^{-\frac{1}{\sigma}} Y_{\kappa(i)}^{\frac{1}{\sigma}-\frac{1}{\epsilon}}, \qquad Y_{\kappa(i)} = \left[\sum_{\{j:\kappa(j)=\kappa(i)\}} y_j^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{50}$$

with $\bar{\omega}_y \equiv Y^{\frac{1}{\epsilon}} P$.

- The optimal pricing equation:

$$p_i = \frac{\mu_i^y}{\mu_i^n} \frac{w_i}{z_i}, \tag{51}$$

  where, $\mu_i^y = \left[\frac{\sigma-1}{\sigma} - \left(\frac{1}{\epsilon} - \frac{1}{\sigma}\right) s_i^y\right]^{-1}$ and $\mu_i^n = \left[\frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n\right]^{-1}$.

- Firm shares are:

$$s_i^y = \frac{y_i^{\frac{\sigma-1}{\sigma}}}{\sum_{j\in\mathcal{M}} y_j^{\frac{\sigma-1}{\sigma}}}, \qquad s_i^n = \frac{n_i^{\frac{1+\eta}{\eta}}}{\sum_{j\in l} n_j^{\frac{\eta+1}{\eta}}}. \tag{52}$$

**Equilibrium Allocation.** Because firm are symmetric within a labor market we have that firms split the market equally,

$$s_i^n = \frac{1}{M_{\ell(i)}}, \quad \ell\left(i\right) \in \left\{l_1, l_2\right\}, \tag{53}$$

and so markdowns are a function of the number of firms in the market only,

$$\mu_i^n = \left[\frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right)\frac{1}{M_i}\right]^{-1}. \tag{54}$$

Absent firm heterogeneity within labor markets the amount of competition in the market is completely capture by the number of competitors in the market. Furthermore, firms within a labor market hire the same number of workers, which simplifies the labor supply equation to,

$$w_i = \bar{\omega}_n \left(n_i\right)^{\frac{1}{\theta}} M_i^{\frac{\eta-\theta}{(1+\eta)\theta}}. \tag{55}$$

Now to get an expression for the sales share we start with the pricing equation, equation (51), and use equation (55) to eliminate firm level wages,

$$p_i = \frac{\mu_i^y}{\mu_i^n}\frac{1}{z_i}\bar{\omega}_n \left(n_i\right)^{\frac{1}{\theta}} M_i^{\frac{\eta}{1+\eta}\left(\frac{1}{\theta}-\frac{1}{\eta}\right)}.$$

Then, we use the production function to eliminate firm level employment and express it in terms of firm level output, and use the inverse demand function to substitute out $p_i$ to get,

$$\bar{\omega}_y y_i^{-\frac{1}{\sigma}} Y_{\kappa(i)}^{\frac{1}{\sigma}-\frac{1}{\epsilon}} = \frac{\mu_i^y}{\mu_i^n}\frac{1}{z_i}\bar{\omega}_n \left(\frac{1}{z_i}y_i\right)^{\frac{1}{\theta}} M_i^{\frac{\eta}{1+\eta}\left(\frac{1}{\theta}-\frac{1}{\eta}\right)}.$$

Solving for $y_i$,

$$y_i = \left(\frac{\mu_i^n}{\mu_i^y}\right)^{\frac{\sigma\theta}{\theta+\sigma}} \left(\frac{\bar{\omega}_n}{\bar{\omega}_y}\right)^{-\frac{\sigma\theta}{\theta+\sigma}} \left(z_i\right)^{\frac{\sigma(\theta+1)}{\theta+\sigma}} M_i^{-\frac{\sigma\theta}{\theta+\sigma}\frac{\eta}{1+\eta}\left(\frac{1}{\theta}-\frac{1}{\eta}\right)} Y_{\kappa(i)}^{\frac{\sigma\theta}{\theta+\sigma}\left(\frac{1}{\sigma}-\frac{1}{\epsilon}\right)}. \tag{56}$$

Finally, we plug equation (56) into equation (52),

$$s_i^y = \frac{\left(\frac{\mu_i^n}{\mu_i^y}\right)^{\frac{(\sigma-1)\theta}{\theta+\sigma}} \left(z_i\right)^{\frac{(\sigma-1)(\theta+1)}{\theta+\sigma}} M_i^{\frac{(\sigma-1)(\eta-\theta)}{(\theta+\sigma)(1+\eta)}}}{\sum_{\{j:\kappa(j)=\kappa(i)\}}\left(\frac{\mu_j^n}{\mu_j^y}\right)^{\frac{(\sigma-1)\theta}{\theta+\sigma}} \left(z_j\right)^{\frac{(\sigma-1)(\theta+1)}{\theta+\sigma}} M_j^{\frac{(\sigma-1)(\eta-\theta)}{(\theta+\sigma)(1+\eta)}}}, \tag{57}$$

with,

$$\mu_i^n = \left[\frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right)\frac{1}{M_i}\right]^{-1}, \quad \text{and} \quad \mu_i^y = \left[\frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right)s_i^y\right]^{-1}. \tag{58}$$

Equations (57) and (58) form a system of non-linear equations that determine the equilibrium values of the sales shares. Although we cannot solve the system with pen and paper, it is possible to determine whether $s_1^y$ is less, bigger or equal than $s_2^y$. We define the function,

$$f(s) \equiv (s)^{\frac{\theta+\sigma}{(\sigma-1)\theta}} \mu^y(s) = \frac{(s)^{\frac{\theta+\sigma}{(\sigma-1)\theta}}}{\frac{\sigma-1}{\sigma} - \left(\frac{1}{\epsilon} - \frac{1}{\sigma}\right) s}.$$

The function $f(s)$ satisfies the properties that $f(0) = 0$, $f(1) = \frac{\epsilon}{\epsilon-1}$, and $f(\cdot)$ is strictly increasing in $s$ ($f'(s) > 0$). From equation (14) we have,

$$f(s_i^y) = \frac{\mu_i^n M_i^{-\frac{(\eta-\theta)}{\theta(1+\eta)}} z_i^{\frac{1+\theta}{\theta}}}{\left[\sum_{j\in\mathcal{M}} \left(\frac{\mu_j^n}{\mu_j^y}\right)^{\frac{(\sigma-1)\theta}{\theta+\sigma}} M_j^{-\frac{(\sigma-1)(\eta-\theta)}{(\theta+\sigma)(1+\eta)}} (z_j)^{\frac{(\sigma-1)(1+\theta)}{\theta+\sigma}}\right]^{\frac{\theta+\sigma}{(\sigma-1)\theta}}},$$

and therefore,

$$\frac{f(s_1^y)}{f(s_2^y)} = \frac{\mu_1^n M_1^{-\frac{(\eta-\theta)}{\theta(1+\eta)}}}{\mu_2^n M_2^{-\frac{(\eta-\theta)}{\theta(1+\eta)}}} \left(\frac{z_1}{z_2}\right)^{\frac{\theta+1}{\theta}}. \tag{59}$$

Define then the function $h(m) = \left(\frac{1}{m}\right)^{\frac{(\eta-\theta)}{\theta(1+\eta)}} \mu_i^n\left(\frac{1}{m}\right)$.This function satisfies that $h(1) = \frac{\theta}{\theta+1}$ , $\lim_{m\to\infty} h(m) = 0$, and $h(\cdot)$ is strictly decreasing in $m$ ($h'(m) < 0$). We can express equation (59) in terms of the function $h(\cdot)$ as,

$$\frac{f(s_1^y)}{f(s_1^y)} = \left(\frac{z_1}{z_2}\right)^{\frac{1+\theta}{\theta}} \frac{h(M_1)}{h(M_2)}.$$

Since $f(\cdot)$ is increasing we know that $s_1^y > s_2^y$ if:

$$\left(\frac{z_1}{z_2}\right)^{\frac{1+\theta}{\theta}} \frac{h(M_1)}{h(M_2)} > 1. \tag{60}$$

Condition (60) is satisfied for different combinations of the parameters. When the two markets have the same amount of competition, i.e. $M_1 = M_2$, then which type of firms gets the largest share is dictated by difference in productivity. Therefore, as in Atkeson and Burstein (2008) and Edmond et al. (2015), the most productive firm gets the largest share. When the two markets differ in the amount of competitions, this relationship can be reverted and the less productive firm could gets

the largest share. Take the case with $z_1 = z_2$, then it follows from $h(\cdot)$ being a decreasing function, that $s_1^y > s_2^y$ if $M_1 < M_2$, and so the the firm in the less competitive labor market captures the largest share despite both firms being equally productive.

**Love for variety effects.** The choice in preferences for labor supply introduces love-for-variety effects into the model.To show that the previous result is not affected by these forces we modify the labor supply as follows:

$$w_i = \bar{\omega}_n (n_i)^{\frac{1}{\eta}} N_{\ell(i)}^{\frac{1}{\theta}-\frac{1}{\eta}}, \quad N_{\ell(i)} = M_{\ell(i)}^{\chi} \left[ \sum_{\{j:\ell(j)=\ell(i)\}} (n_j)^{\frac{\eta+1}{\eta}} \right]^{\frac{\eta}{1+\eta}}.$$

In the standard set-up the parameter $\eta$ controls both the elasticity of substitution across firms and the love-for-variety effects in the model[47]. By introducing $\chi$ into the model the parameter $\eta$ controls the former effect but not the latter. The parameter $\chi$ controls the love-for-variety effects, when $\chi = \frac{1}{1+\eta}$ there are no love-for-variety effects. When $\chi = 0$ we go back to the original model.

Now, in the symmetric equilibrium, we have that $N_{\ell(i)} = M_{\ell(i)}^{\chi+\frac{\eta}{\eta+1}} n_i$ , and therefore sales share satisfy the equation:

$$\frac{f(s_1^y)}{f(s_1^y)} = \left(\frac{z_1}{z_2}\right)^{\frac{1+\theta}{\theta}} \frac{h_\chi(M_1)}{h_\chi(M_2)}, \tag{61}$$

where

$$h_\chi(m) = m^{-\left(\chi+\frac{\eta}{1+\eta}\right)\left(\frac{1}{\theta}-\frac{1}{\eta}\right)} \mu_i^n \left(\frac{1}{m}\right).$$

For the case with no love-for-variety effects, i.e. $\chi = \frac{1}{1+\eta}$, we still have that: $h_{\frac{1}{1+\eta}}(1) = \frac{\theta}{\theta+1}$ , $\lim_{m\to\infty} h_{\frac{1}{1+\eta}}(m) = 0$, and $h_{\frac{1}{1+\eta}}(\cdot)$ is strictly decreasing in $m$ . Therefore, the previous analysis does not change once we remove the love-for-variety effects from the mode.

# B Identification of Gains From Trade

In this section we describe how to compute the pro-competitive gains from trade. As we mentioned in the main text the key to compute the pro-competitive gains from trade is to be able to compute

[47]For more on the topic see Benassy (1998)

the counterfactual autarky equilibrium given the observed trade equilibrium. The main discussion in this section is then about the autarky equilibrium. In this section we assume that we know the values of the parameters $\{\eta, \theta, \sigma, \epsilon, \varphi, \bar{\varphi}\}$, the partition of the economy into labor and product markets and the assignment functions $\kappa(\cdot)$ and $\ell(\cdot)$, and so this section is about identifying the shifters and the productivities of the firm in Home economy.

In terms of data we assume we observed firm level wages, domestic sales, exports and payroll. At the aggregate level we have information on sectoral imports and gross output. Moving forward is important to define some notation:

- We use $\{M_k\}_{k\in\mathcal{K}}$ to denote the total expenditure in imported inputs of $k$-type products .
- We use $s_i^n$ to denote the payroll share of firm $i$ and $w_i$ the wage it pays its workers in the equilibrium.
- We use $s_{H,i}^y$ to denote the sales share of firm $i$ in its domestic market in the equilibrium, and $d_i$ for the value of those domestic sales.
- We use $S_k^y$ to denote the expenditure share in goods from product market $k$, this includes both domestic sales and imports.

## B.1 Part I: The synthetic economy.

As reminder, the sysnthetic economy has a different set of parameters as the original economy, but rationalizes the same data and prices in equilibrium equal one. The parameters of this economy are given by,

$$
\begin{aligned}
\xi_{n,i}^{se} &= \frac{s_i^n}{\left[\sum_{\{j:\ell(j)=\ell(i)\}}\left(\frac{w_i}{w_j}\right)^{1+\eta} s_j^n\right]}, \qquad \forall i \in \mathcal{M}, \\
\xi_{Hy,i}^{se} &= s_{H,i}^y, \qquad \forall i \in \mathcal{M}, \\
z_i^{se} &= \frac{\mu_i^y\left(s_{Hi}^y\right)}{\mu_i^n\left(s_i^n\right)} w_i, \qquad \forall i \in \mathcal{M}, \\
\upsilon_{n,l}^{se} &= \frac{S_l^n / W_l^{1+\theta}}{\sum_{\{l':\ell(j)=l\}} S_{l'}^n / W_{l'}^{1+\theta}}, \quad \forall l \in \mathcal{L}, \\
\upsilon_{y,k}^{se} &= S_k^y, \qquad \forall k \in \mathcal{K}.
\end{aligned}
$$

We note that, $\xi^{se}_{n,i} = \xi_{n,i}$ and $\upsilon^{se}_{n,l} = \upsilon_{n,l}$ since both economy rationalizes the same firm-level wages and payroll shares.

## B.2 Part II: Counterfactual equilibrium equivalence.

Since we know the fundamentals in the autarky equilibrium we also know the autarky equilibrium in the synthetic economy. The next step is to show how this synthetic economy inform us about the autarky equilibrium of the true economy, which is the object we are interested in computing. The result is summarize in the following proposition were we used $\tilde{x}$ to denote variables in the autarky equilibrium.

**Proposition B.1.** If $\left\{\tilde{w}^{se}_i, \tilde{p}^{se}_i, \tilde{s}^{n,se}_i, \tilde{s}^{y,se}_{H,i}\right\}_{i\in\mathcal{M}}$ is part of an autarky equilibrium in our artificial $se$ economy, then, $\left\{\tilde{w}^{se}_i, \frac{z^{se}_i}{z_i}\tilde{p}^{se}_i, \tilde{s}^{n,se}_i, \tilde{s}^{y,se}_{H,i}\right\}_{i\in\mathcal{M}}$ is an equilibrium under autarky of the true economy. Furthermore, the synthetic economy and the true economy have the same welfare level in the autarky equilibrium.

*Proof.* We'll prove this result in several steps, showing relationships between the quantities in the two economies in the autarky equilibrium and then using this relationship to conclude the proof.

**Claim 1: If $\tilde{w}_i = \tilde{w}^{se}_i$, then $\tilde{s}^{n}_i = \tilde{s}^{n,se}_i$.**

To show this we start with our equation for payroll shares in the true economy. These shares need to satisfy,

$$\begin{aligned}
\tilde{s}^{n}_i &= \xi_{n,i}\frac{\left(\tilde{w}_i\right)^{1+\eta}}{\sum_{\{j\in\mathcal{M}:\ell(j)=\ell(i)\}}\xi_{n,j}\left(\tilde{w}_j\right)^{1+\eta}} \\
&= \xi_{n,i}\frac{\left(\tilde{w}^{se}_i\right)^{1+\eta}}{\sum_{\{j\in\mathcal{M}:\ell(j)=\ell(i)\}}\xi_{n,j}\left(\tilde{w}^{se}_j\right)^{1+\eta}} \\
&= \xi^{se}_{n,i}\frac{\left(\tilde{w}^{se}_i\right)^{1+\eta}}{\sum_{\{j\in\mathcal{M}:\ell(j)=\ell(i)\}}\xi^{se}_{n,j}\left(\tilde{w}^{se}_j\right)^{1+\eta}} \\
&= \tilde{s}^{n,se}_i,
\end{aligned}$$

where we used that $\tilde{w}_i = \tilde{w}^{se}_i$ and $\xi_{n,i} = \xi^{se}_{n,i}$ in the second and third equality. Note that is trivial to show that the following is also true between the two economies,

$$\tilde{W}^{se}_l = \tilde{W}_l, \quad \forall l \in \mathcal{L},$$

and therefore,

$$\tilde{W}^{se} = \tilde{W}.$$

**Claim 2: If $\tilde{w}_i = \tilde{w}_i^{se}$ , then $\tilde{p}_{H,i} = \tilde{p}_{H,i}^{se} p_{H,i}$.**

Prices need to satisfy the first order condition of the firm. Hence,

$$\tilde{p}_{H,i} = \frac{\mu^y\left(\tilde{s}_{H,i}^y\right)}{\mu^n\left(\tilde{s}_{H,i}^n\right)} \frac{\tilde{w}_i}{z_i}.$$

Therefore,

$$\begin{aligned}\frac{\tilde{p}_{H,i}}{\tilde{p}_{H,i}^{se}} &= \frac{\mu^y\left(\tilde{s}_{H,i}^y\right)}{\mu^n\left(\tilde{s}_{H,i}^n\right)} \frac{\tilde{w}_i}{z_i} \frac{1}{\frac{\mu^y\left(\tilde{s}_{H,i}^{y,se}\right)}{\mu^n\left(\tilde{s}_{H,i}^{n,se}\right)} \frac{\tilde{w}_i^{se}}{z_i^{se}}} \\ &= \frac{\mu^y\left(\tilde{s}_{H,i}^y\right)}{\mu^y\left(\tilde{s}_{H,i}^{y,se}\right)} \frac{z_i^{se}}{z_i}.\end{aligned}$$

Now, since the synthetic and the true economy rationalize the same set of observables we have,

$$z_i = \frac{w_i}{p_{H,i}} \frac{\mu^y\left(s_{H,i}^y\right)}{\mu^n\left(s_i^n\right)}, \quad z_i^{se} = w_i \frac{\mu^y\left(s_{H,i}^y\right)}{\mu^n\left(s_i^n\right)},$$

and so

$$\frac{\tilde{p}_{H,i}}{\tilde{p}_{H,i}^{se}} = \frac{\mu^y\left(\tilde{s}_{H,i}^y\right)}{\mu^y\left(\tilde{s}_{H,i}^{y,se}\right)} p_{H,i}.$$

We need to show now that for all $i \in \mathcal{M}$, $\tilde{s}_{H,i}^{y,se} = \tilde{s}_{H,i}^y$. To do this we start with the equation for sales share,

$$\begin{aligned}\tilde{s}_{H,i}^y &= \frac{\xi_{Hy,i}\left(\tilde{p}_{H,i}\right)^{1-\sigma}}{\sum_{\{j \in \mathcal{M}:\kappa(j)=k\}} \xi_{Hy,j}\left(\tilde{p}_{H,j}\right)^{1-\sigma}} \\ &= \frac{\xi_{Hy,i}\left(\frac{\mu^y\left(\tilde{s}_{H,i}^y\right)}{\mu^y\left(\tilde{s}_{H,i}^{y,se}\right)} \tilde{p}_{H,i}^{se} p_{H,i}\right)^{1-\sigma}}{\sum_{\{j \in \mathcal{M}:\kappa(j)=k\}} \xi_{Hy,j}\left(\frac{\mu^y\left(\tilde{s}_{H,j}^y\right)}{\mu^y\left(\tilde{s}_{H,j}^{y,se}\right)} \tilde{p}_{H,j}^{se} p_{H,j}\right)^{1-\sigma}}.\end{aligned}$$

At the initial equilibrium we know that, $\xi_{Hy,i}\left(p_{H,i}\right)^{1-\sigma}=s_{H,i}^{y}\left(P_{k}\right)^{1-\sigma}$. Hence,

$$\tilde{s}_{H,i}^{y}=\frac{s_{H,i}^{y}\left(\frac{\mu^{y}\left(\tilde{s}_{H,i}^{y}\right)}{\mu^{y}\left(\tilde{s}_{H,i}^{y,se}\right)}\tilde{p}_{H,i}^{se}\right)^{1-\sigma}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=k\}}s_{H,j}^{y}\left(\frac{\mu^{y}\left(\tilde{s}_{H,j}^{y}\right)}{\mu^{y}\left(\tilde{s}_{H,j}^{y,se}\right)}\tilde{p}_{H,j}^{se}\right)^{1-\sigma}}. \tag{62}$$

From the synthetic economy we have a similar equation,

$$\tilde{s}_{H,i}^{y,se}=\frac{\xi_{Hy,i}^{se}\left(\tilde{p}_{H,i}^{se}\right)^{1-\sigma}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=k\}}\xi_{Hy,j}^{se}\left(\tilde{p}_{H,j}^{se}\right)^{1-\sigma}},\quad \xi_{Hy,i}^{se}=s_{H,i}^{y},$$

or

$$\tilde{s}_{H,i}^{y,se}=\frac{s_{H,i}^{y}\left(\tilde{p}_{H,i}^{se}\right)^{1-\sigma}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=k\}}s_{H,i}^{y}\left(\tilde{p}_{H,j}^{se}\right)^{1-\sigma}}. \tag{63}$$

Then, $\left\{\tilde{s}_{H,i}^{y,se},\tilde{s}_{H,i}^{y}\right\}$ need to satisfy equations (62) and (63). It is straightforward to verify that $\tilde{s}_{H,i}^{y,se}=\tilde{s}_{H,i}^{y}$ satisfies the system of equation. It follows then that $\tilde{p}_{H,i}=\tilde{p}_{H,i}^{se}p_{H,i}$ and so $\tilde{p}_{H,i}^{se}$ give us the change in firm-level prices from the true to the autarky equilibrium. Note that this also implies that, $\tilde{p}_{H,i}=\frac{z_{i}^{se}}{z_{i}}\tilde{p}_{H,i}^{se}$.

**Claim 3: If $\tilde{w}_{i}=\tilde{w}_{i}^{se}$, then $\tilde{P}_{k}=\tilde{P}_{k}^{se}P_{k}$ and $\tilde{P}=\tilde{P}^{se}P$.**

We start from product $k$ price index equation,

$$\begin{aligned}
\tilde{P}_{k} &= \left[\sum_{\{i\in\mathcal{M}:\kappa(i)=k\}}\xi_{Hy,i}\left(\tilde{p}_{H,i}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \\
&= \left[\sum_{\{i\in\mathcal{M}:\kappa(i)=k\}}\xi_{Hy,i}\left(p_{H,i}\right)^{1-\sigma}\left(\tilde{p}_{H,i}^{se}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \\
&= \left[\sum_{\{i\in\mathcal{M}:\kappa(i)=k\}}s_{H,i}^{y}\left(\tilde{p}_{H,i}^{se}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}P_{k} \\
&= \left[\sum_{\{i\in\mathcal{M}:\kappa(i)=k\}}\xi_{Hy,i}^{se}\left(\tilde{p}_{H,i}^{se}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}P_{k} \\
&= \tilde{P}_{k}^{se}P_{k}.
\end{aligned}$$

Similarly, starting for the formula for the price index of the economy,

$$\begin{aligned}
\tilde{P} &= \left[\sum_{k\in\mathcal{K}} \upsilon_{y,k}\tilde{P}_k^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \\
&= \left[\sum_{k\in\mathcal{K}} \upsilon_{Hy,k}\left(\tilde{P}_k^{se}P_k\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \\
&= \left[\sum_{k\in\mathcal{K}} S_k^{y}\left(\tilde{P}_k^{se}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} P \\
&= \left[\sum_{k\in\mathcal{K}} \upsilon_{y,k}^{se}\left(\tilde{P}_k^{se}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} P \\
&= \tilde{P}^{se}P.
\end{aligned}$$

It follows then that $\tilde{P}_k = \tilde{P}_k^{se}P_k$ and so $\tilde{P}_k^{se}$ give us the change in market-level prices from the true to the autarky equilibrium. similarly, $\tilde{P}^{se}$ give us the change in the aggregate price index. Since we normalized $P = 1$ this result also implies that $\tilde{P} = \tilde{P}^{se}$.

**Claim 4: If $\tilde{w}_i = \tilde{w}_i^{se}$ then $\tilde{\pi}_i = \tilde{\pi}_i^{se}\frac{\tilde{N}}{\tilde{N}^{se}}$.**

We noticed that in either of the two equilibriums, labor supply and product demand can be written as:

$$n_i = \xi_{ni}\upsilon_{n\ell(i)}\left(\frac{s_i^n}{\xi_{ni}}\right)^{\frac{\eta}{1+\eta}}\left(\frac{S_{\ell(i)}^n}{\upsilon_{n,\ell(i)}}\right)^{\frac{\theta}{1+\theta}} N,$$

$$p_{Hi} = \left(\frac{s_i^y}{\xi_{Hyi}}\right)^{\frac{1}{1-\sigma}}\left(\frac{S_{\kappa(i)}^y}{\upsilon_{y\kappa(i)}}\right)^{\frac{1}{1-\epsilon}}.$$

With these two equations we can now write firm profits as:

$$\begin{aligned}
\pi_i &= (p_{Hi} z_i - w_i)\, n_i \\
&= z_i p_{Hi} \left(1 - \frac{\mu^n\left(s_i^n\right)}{\mu^y\left(s_{H,i}^y\right)}\right) n_i \\
&= z_i \left(\frac{s_i^y}{\xi_{Hyi}}\right)^{\frac{1}{1-\sigma}} \left(\frac{S_{\kappa(i)}^y}{\upsilon_{y\kappa(i)}}\right)^{\frac{1}{1-\epsilon}} \left(1 - \frac{\mu^n\left(s_i^n\right)}{\mu^y\left(s_{H,i}^y\right)}\right) \xi_{ni} \upsilon_{n\ell(i)} \left(\frac{s_i^n}{\xi_{ni}}\right)^{\frac{\eta}{1+\eta}} \left(\frac{S_{\ell(i)}^n}{\upsilon_{n,\ell(i)}}\right)^{\frac{\theta}{1+\theta}} N \\
&= \pi_i\left(s_i^n, S_{\ell(i)}^n, s_{Hi}^y, S_{\kappa(i)}^y\right) N.
\end{aligned}$$

So far we showed that if $\tilde{w}_i = \tilde{w}_i^{se}$, then $\tilde{s}_i^n = \tilde{s}_i^{nse}$ and $\tilde{S}_l^n = \tilde{S}_{\ell(i)}^{se,n}$ (claim 1), $\tilde{s}_{Hi}^y = \tilde{s}_{Hi}^{se,y}$ and $\tilde{S}_k^y = \tilde{S}_k^{se,y}$(claim 2). Therefore,

$$\tilde{\pi}_i = \tilde{\pi}_i^{se} \frac{\tilde{N}}{\tilde{N}^{se}}.$$

This implies,

$$\tilde{\Pi} = \tilde{\Pi}^{se} \frac{\tilde{N}}{\tilde{N}^{se}}.$$

**Claim 5: If $\tilde{w}_i = \tilde{w}_i^{se}$ then $\tilde{C} = \tilde{C}^{se}$ and $\tilde{N} = \tilde{N}^{se}$.**

The budget constraint give us the relationship:

$$C = WN + \Pi,$$

and from Claim 4 we have:

$$\tilde{C} = \left(\tilde{W} + \tilde{\Pi}\left(\tilde{s}^n, \tilde{S}^n, \tilde{s}^y, \tilde{S}^y\right)\right) \tilde{N}, \quad \tilde{C}^{se} = \left(\tilde{W} + \tilde{\Pi}\left(\tilde{s}^n, \tilde{S}^n, \tilde{s}^y, \tilde{S}^y\right)\right) \tilde{N}^{se},$$

and so

$$\frac{\tilde{C}}{\tilde{C}^{se}} = \frac{\tilde{N}}{\tilde{N}^{se}}.$$

The second equation is the first order condition between aggregate consumption and aggregate labor. When utility is linear in consumption this equation is $\bar{\varphi}^{\frac{1}{\varphi}} N^{\frac{1}{\varphi}} = W$. Since both economy have the same level of aggregate wages it is straightforward to show that $\tilde{N} = \tilde{N}^{se}$ and $\tilde{C} = \tilde{C}^{se}$. With log

utility in consumption the relevant equation is $\bar{\varphi}^{\frac{1}{\varphi}} N^{\frac{1}{\varphi}} C = W$. In this case we get the system

$$\tilde{N}^{\frac{1}{\varphi}} \tilde{C} = \left(\tilde{N}^{se}\right)^{\frac{1}{\varphi}} \tilde{C}^{se}, \qquad \frac{\tilde{C}}{\tilde{C}^{se}} = \frac{\tilde{N}}{\tilde{N}^{se}}.$$

The only solution for this is again $\tilde{N} = \tilde{N}^{se}$ and $\tilde{C} = \tilde{C}^{se}$.

**Claim 6: If $\tilde{w}_i = \tilde{w}_i^{se}$, then $\tilde{n}_j = \tilde{n}_j^{se}$ and $\tilde{\pi}_i = \tilde{\pi}_i^{se}$.**

This follows from Claim 4 and Claim 5.

**Claim 7: If $\tilde{w}_i = \tilde{w}_i^{se}$ then $\tilde{y}_{H,i} = \frac{z_i}{z_i^{se}} \tilde{y}_{H,i}^{se}$.**

We start from the demand for inputs and use the previous claims to get,

$$\begin{aligned}
\tilde{y}_{H,i} &= \xi_{Hy,i} \upsilon_{y,k} \left(\frac{\tilde{p}_{H,i}}{\tilde{P}_k}\right)^{-\sigma} \left(\frac{\tilde{P}_k}{\tilde{P}}\right)^{-\epsilon} \tilde{Y} \\
&= \xi_{Hy,i} \upsilon_{y,k} \left(\frac{\tilde{p}_{H,i}^{se} p_{H,i}}{\tilde{P}_k^{se} P_k}\right)^{-\sigma} \left(\frac{\tilde{P}_k^{se} P_k}{\tilde{P}^{se}}\right)^{-\epsilon} \tilde{Y} \\
&= \frac{1}{s_{H,i}^y} \frac{1}{S_k^y} \frac{1}{Y} y_{Hi} \tilde{y}_{H,i}^{se} \\
&= \frac{1}{p_{Hi}} \tilde{y}_{H,i}^{se}.
\end{aligned}$$

Now from the pricing equation at the original equilibrium we have,

$$p_{H,i} = \frac{\mu^y\left(s_{H,i}^y\right)}{\mu^n\left(s_i^n\right)} \frac{w_i}{z_i} = \frac{z_i^{se}}{z_i} \frac{\mu^y\left(s_{H,i}^y\right)}{\mu^n\left(s_i^n\right)} \frac{w_i}{z_i^{se}} = \frac{z_i^{se}}{z_i}.$$

Putting the two equations together yields the result.

Putting together all this results we can now prove the result. We start with $\left\{\tilde{w}_i^{se}, \tilde{p}_i^{se}, \tilde{y}_{H,i}^{se}, \tilde{n}_i^{se}\right\}_{i \in \mathcal{M}}$ being part of an equilibrium in the *se* economy. Then $\left\{\tilde{w}_i^{se}, \frac{z_i^{se}}{z_i} \tilde{p}_i^{se}, \frac{z_i}{z_i^{se}} \tilde{y}_{H,i}^{se}, \tilde{n}_i^{se}\right\}_{i \in \mathcal{M}}$ is an equilibrium under autarky of the true economy:

- $\left\{\tilde{w}_i^{se}, \frac{z_i^{se}}{z_i} \tilde{p}_i^{se}\right\}_{i \in \mathcal{M}}$ satisfy the pricing equation.
- Given $\left\{\tilde{w}_i^{se}, \tilde{\Pi}^{se}\right\}_{i \in \mathcal{M}}$ , $\{\tilde{n}_i^{se}\}_{i \in \mathcal{M}}$ and $\tilde{C}^{se}$ is the utility maximizing choice of the household.
- Given $\left\{\frac{z_i^{se}}{z_i} \tilde{p}_i^{se}\right\}_{i \in \mathcal{M}}$, $\left\{\frac{z_i}{z_i^{se}} \tilde{y}_{H,i}^{se}\right\}_{i \in \mathcal{M}}$ is the profit maximizing choice of the final good consumer.
- The allocation is feasible.

- Since $\tilde{C} = \tilde{C}^{se}$ and $\tilde{N} = \tilde{N}^{se}$, the two equilibrium have the same utility levels.

Furthermore, the equilibrium of the synthetic economy has the same payroll and sales shares as the true economy. □

Proposition B.1 tell us that we can use the synthetic economy to construct the autarky values of firm-level shares, market-level shares and welfare. The next result proves the identification of welfare gains and the change in the distribution of market power.

**Proposition B.2.** Given the elasticity parameters $(\sigma, \epsilon, \eta, \theta, \varphi, \bar{\varphi})$, assignment functions $\kappa(\cdot)$ and $\ell(\cdot)$, and data on (i) firm level wages, (ii) firm-level payroll and domestic sales, and (iii) and expenditure shares on sectoral imports, we can identified the gains from trade of moving from the observed trade equilibrium to the autarky equilibrium is identified, and the change in the distribution of firm-level market power.

*Proof.* From Proposition B.1 we know how to compute welfare and firm shares in the autarky equilibrium. Because the shares are observed in the trade equilibrium it follows that the change in firm market power is identified.We also know welfare in autarky equilibrium, so the only thing that remains to be shown is how to compute welfare in the observed trade equilibrium.

Welfare in the trade equilibrium is given by:

$$U = \begin{cases} C - \bar{\varphi}^{\frac{1}{\varphi}} \frac{1}{1+\frac{1}{\varphi}} N^{1+\frac{1}{\varphi}} & \text{linear utility in consumption} \\ \log C - \bar{\varphi}^{\frac{1}{\varphi}} \frac{1}{1+\frac{1}{\varphi}} N^{1+\frac{1}{\varphi}} & \text{log utility in consumption} \end{cases},$$

where in both cases aggregate consumption satisfies:

$$C = Y = \sum_{i \in \mathcal{M}} p_{H,i} y_{H,i} + \sum_{i \in \mathcal{M}^*} p_{F,i} y_{F,i} = \sum_{i \in \mathcal{M}} d_i^y + \sum_{k \in \mathcal{K}} M_k,$$

that is the sum of total domestic sales plus total imports. Aggregate employment is given instead by:

$$N = \begin{cases} \frac{1}{\bar{\varphi}} W^{\varphi} & \text{linear utility in consumption} \\ \frac{1}{\bar{\varphi}} \left(\frac{W}{C}\right)^{\varphi} & \text{log utility in consumption} \end{cases},$$

where in both cases we can compute $W$ from the information on firm wages. □

# C Algorithm to solve the Closed-economy version

In this section: (i) we described the closed economy version of the mode; and (ii) described the fixed point algorithm used to solve for the autarky equilibrium of the model.

## C.1 Closed Economy

### C.1.1 Environment and Optimal Choices

**Final Good Producer.** Relative to the open economy model the main distinction here is that the production function of the final good producer uses only intermediate goods produced in the domestic economy. Total production is then,

$$Y = \left[\sum_{k\in\mathcal{K}} \upsilon_{y,k}^{\frac{1}{\epsilon}} Y_k^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}},$$

where now $Y_k$ is given by,

$$Y_k = \left[\sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} \xi_{y,i}^{\frac{1}{\sigma}} y_i^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

The demand for each individual variety is then described by the equation,

$$y_i = \xi_{y,i}\left(\frac{p_i}{P_{\kappa(i)}}\right)^{-\sigma} Y_{\kappa(i)}, \quad Y_{\kappa(i)} = \upsilon_{y,\kappa(i)}\left(\frac{P_{\kappa(i)}}{P}\right)^{-\epsilon} Y, \tag{64}$$

where $p_i$ is the unit price of variety $i$, and $P_{\kappa(i)}$ and $P$ are the market and aggregate price indexes given by,

$$P_k = \left[\sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} \xi_{y,i} p_i^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \quad \text{and} \quad P = \left[\sum_{k\in\mathcal{K}} \upsilon_{y,k} P_k^{1-\epsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{65}$$

**Preferences.** The labor supply decision of the household and demand for aggregate consumption

are still given by equations (3), (2), and (5),

$$
\begin{aligned}
w_i &= \left(\frac{n_i}{N_{\ell(i)}}\right)^{\frac{1}{\eta}} \left(\frac{N_{\ell(i)}}{N}\right)^{\frac{1}{\theta}} W, \\
W_{\ell(i)}^{1+\eta} &= \sum_{\{j \in \mathcal{M}: \ell(j)=\ell(i)\}} \xi_{n,j} w_j^{1+\eta}, \\
W^{1+\theta} &= \sum_{l \in \mathcal{L}} \upsilon_{n,l} W_l^{1+\eta}, \\
N &= \frac{1}{\bar{\varphi}} \left(\frac{W}{P}\right)^{\varphi}.
\end{aligned} \tag{66}
$$

**Intermediate Good producer.** These firms engage in imperfect competition both in the product and labor market playing a static game of quantity competition. All of the firm's competitors are located in the domestic economy since trade is not allow. Each firm $i$ chooses its quantity $y_i$ and its employment level $n_i$, taking as given the quantities and employment decisions made by the other firms in the economy, but recognizing that market prices and wages vary with the firm's decision. Then, an intermediate good producer $i$ operating in labor market $\ell(i) = l$ and selling its good in product market $\kappa(i) = k$ chooses quantity $y_i$ and employment $n_i$ to maximize profits. The optimal price and wage charged by the firm are related to each other according to the equation,

$$
p_i = \mu_i^y \times \frac{1}{\mu_i^n} \frac{w_i}{z_i},
$$

where $\mu_i^y$ and $\mu_i^n$ are the markup and the markdown of the firm, respectively, given by the equations,

$$
\frac{1}{\mu_i^y} = \left[\frac{\sigma - 1}{\sigma} - \left(\frac{1}{\epsilon} - \frac{1}{\sigma}\right) s_i^y\right]^{-1}, \quad \frac{1}{\mu_i^n} = \left[\frac{\eta + 1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n\right],
$$

where $s_i^y$ and $s_i^n$ are the sales and payroll share of the firm in the product and labor market where she operates.

### C.1.2 Equilibrium in term of firm and market level shares

In this section we show how to solved for the Autarky equilibrium of the model. We start with a definition of the equilibrium and then show that it can be simplified by expressing it as function of firm and market level shares only. This transformation does not reduce the dimension of the

system, however it is useful for two reasons. First, it allows for a nested fixed point algorithm. where the inner nest solves for firm level shares for given values of market level shares. The outer nest uses the firm level shares from the previous step to construct the equilibrium value of output and employment for each firm and updates the values of the sales and payroll shares at the market level. The second reason why this transformation is useful is because sales and payroll shares are by definition between zero and one and this provides stability to the nested fixed point algorithm.

### Autarky equilibrium

**Definition C.1.** The equilibrium of the closed economy consist of the following: a set of firm level prices $\{p_i\}_{i\in\mathcal{M}}$ and wages $\{w_i\}_{i\in\mathcal{M}}$; a set of price indexes for each product market $\{P_k\}_{k\in\mathcal{K}}$ and a set of wage indexes for each labor market $\{W_l\}_{l\in\mathcal{L}}$; an aggregate price and wage index $\{P, W\}$ and aggregate profits $\Pi$; a set of firm level quantities $\{y_i\}_{i\in\mathcal{M}}$ and employment $\{n_i\}_{i\in\mathcal{M}}$, and aggregate level of consumption of the final good $Y$ and employment $N$ such that:

- Given prices, wages and aggregate profits, the quantities $\{n_i, Y\}$ maximize the utility of the household.
- Given prices, the quantities $\{y_i, Y\}$ maximize profits for the final good producer.
- Given aggregates $\{W, P, N, Y\}$ , $\{n_i, y_i, p_i, w_i\}$ maximize profits of the intermediate good producer.
- All product and labor markets clear.

### A recursion for firm level shares

In this section, we derived the system of equations that has as solution firm level sales and payroll shares $\{s_i^y, s_i^n\}_{i\in\mathcal{M}}$ as functions of market level sales and payroll shares $\{\{S_k^y\}_{k\in\mathcal{K}}, \{S_l^n\}_{l\in\mathcal{L}}\}$. The market level shares are defined analogously to their firm level counterparts,

$$S_k^y \equiv \frac{P_k Y_k}{\sum_{k'\in\mathcal{K}} P_{k'} Y_{k'}}, \qquad S_l^n \equiv \frac{W_l N_l}{\sum_{l'\in\mathcal{L}} W_{l'} N_{l'}}.$$

We start with the following set of equations for the firm and market level shares that are standard in model with CES structures,

$$s_i^y = \xi_{y,i}\left(\frac{p_i}{P_{\kappa(i)}}\right)^{1-\sigma}, \qquad \text{and} \quad s_i^n = \xi_{n,i}\left(\frac{w_i}{W_{\ell(i)}}\right)^{1+\eta}, \tag{67}$$

where $P_{\kappa(i)}$ and $W_{\ell(i)}$ are defined in equations (65) and (66). To derive our expression for the firm level shares, we combine equation (67) with: (i) the production technology of the firm; (ii) firm $i$'s labor supply; (iii) firm $i$'s inverse demand function; and (iv) the optimal pricing equation of the firm. These are given by,

$$\begin{aligned}
y_i &= z_i n_i, \\
n_i &= \xi_{n,i}\left(\frac{w_i}{W_{\ell(i)}}\right)^{\eta} N_{\ell(i)}, \\
p_i &= \left(\frac{1}{\xi_{y,i}}\frac{y_i}{Y_{\kappa(i)}}\right)^{-\frac{1}{\sigma}} P_{\kappa(i)}, \\
p_i &= \frac{\mu_i^y}{\mu_i^n}\frac{w_i}{z_i}.
\end{aligned}$$

**Step 1: An expression for firm level prices and wages.** Starting with the inverse demand function, we can use the production function to eliminate $y_i$ and then the labor supply equation to substitute out $n_i$,

$$p_i = \left(\frac{z_i\xi_{n,i}}{\xi_{y,i}}\right)^{-\frac{1}{\sigma}} (w_i)^{-\frac{\eta}{\sigma}} \left(\frac{W_{\ell(i)}N_{\ell(i)}}{\left(W_{\ell(i)}\right)^{1+\eta}}\frac{\left(P_{\kappa(i)}\right)^{1-\sigma}}{P_{\kappa(i)}Y_{\kappa(i)}}\right)^{-\frac{1}{\sigma}}.$$

Using the pricing equation to substitute out firm level wages $w_i$ and solving for prices $p_i$ yields,

$$p_i = \left(\frac{\xi_{y,i}}{\xi_{n,i}}\right)^{\frac{1}{\sigma+\eta}} (z_i)^{-\frac{1+\eta}{\sigma+\eta}} \left(\frac{\mu_i^n}{\mu_i^y}\right)^{-\frac{\eta}{\sigma+\eta}} \left[\frac{P_{\kappa(i)}Y_{\kappa(i)}/P_{\kappa(i)}^{1-\sigma}}{W_{\ell(i)}N_{\ell(i)}/W_{\ell(i)}^{1+\eta}}\right]^{\frac{1}{\sigma+\eta}}, \tag{68}$$

which implies that firm level wages are,

$$w_i = \left(\frac{\xi_{y,i}}{\xi_{n,i}}\right)^{\frac{1}{\sigma+\eta}} (z_i)^{\frac{\sigma-1}{\sigma+\eta}} \left(\frac{\mu_i^n}{\mu_i^y}\right)^{\frac{\sigma}{\sigma+\eta}} \left[\frac{P_{\kappa(i)}Y_{\kappa(i)}/P_{\kappa(i)}^{1-\sigma}}{W_{\ell(i)}N_{\ell(i)}/W_{\ell(i)}^{1+\eta}}\right]^{\frac{1}{\sigma+\eta}}. \tag{69}$$

**Step 2: Expressing market level variables in terms of market level shares.** The next step is to eliminate market level variables using instead market level shares and aggregate quantities. For this we use again that the CES structure in preference allow us to express market level quantities and market level shares as,

$$Y_k = \upsilon_{y,k}\left(\frac{P_k}{P}\right)^{-\epsilon} Y, \qquad S_k^y \equiv \frac{P_k Y_k}{PY} = \upsilon_{y,k}\left(\frac{P_k}{P}\right)^{1-\epsilon} \quad \text{for all } k \in \mathcal{K}.$$

Therefore,

$$\frac{P_{\kappa(i)}Y_{\kappa(i)}}{P_{\kappa(i)}^{1-\sigma_{\kappa(i)}}} = S_{\kappa(i)}^y \frac{PY}{P_{\kappa(i)}^{1-\sigma}} = \upsilon_{y,\kappa(i)}\left(\frac{S_{\kappa(i)}^y}{\upsilon_{y,\kappa(i)}}\right)^{\frac{\sigma-\epsilon}{1-\epsilon}} \frac{PY}{P^{1-\sigma}}. \tag{70}$$

A similar relationship between market level quantities and market level shares holds in labor markets and therefore we also have that,

$$\frac{N_{\ell(i)}W_{\ell(i)}}{W_{\ell(i)}^{1+\eta}} = \upsilon_{n,\ell(i)}\left(\frac{S_{\ell(i)}^n}{\upsilon_{n,\ell(i)}}\right)^{\frac{\theta-\eta}{1+\theta}} \frac{WN}{W^{1+\eta}}. \tag{71}$$

**Step 3: Compute firm level shares.** The last step involves substituting equations (68) to (71) into (67). Firm level payroll shares are,

$$s_i^n = \frac{\xi_{n,i}\left(\frac{\xi_{y,i}}{\xi_{n,i}}\right)^{\frac{1+\eta}{\sigma+\eta}}\left(\upsilon_{y,\kappa(i)}\right)^{\frac{(\sigma-1)(1+\eta)}{(\epsilon-1)(\sigma+\eta)}}(z_i)^{\frac{(\sigma-1)(1+\eta)}{\sigma+\eta}}\left(\frac{\mu_i^n}{\mu_i^y}\right)^{\frac{\sigma(1+\eta)}{\sigma+\eta}}\left(S_{\kappa(i)}^y\right)^{\frac{(\sigma-\epsilon)(1+\eta)}{(1-\epsilon)(\sigma+\eta)}}}{\sum_{\{j\in\mathcal{M}:\ell(j)=\ell(i)\}}\xi_{n,j}\left(\frac{\xi_{y,j}}{\xi_{n,j}}\right)^{\frac{1+\eta}{\sigma+\eta}}\left(\upsilon_{y,\kappa(j)}\right)^{\frac{(\sigma-1)(1+\eta)}{(\epsilon-1)(\sigma+\eta)}}(z_j)^{\frac{(\sigma-1)(1+\eta)}{\sigma+\eta}}\left(\frac{\mu_j^n}{\mu_j^y}\right)^{\frac{\sigma(1+\eta)}{\sigma+\eta}}\left(S_{\kappa(j)}^y\right)^{\frac{(\sigma-\epsilon)(1+\eta)}{(1-\epsilon)(\sigma+\eta)}}}. \tag{72}$$

Note that the denominator sums across all firm $j \in \mathcal{M}$ that share the labor market with firm $i$ and therefore labor market aggregates cancel out. Furthermore, aggregate level variables also cancel out since those affect all firm in the same way. Finally, product market aggregates do not cancel out from our expression since firms in a labor market might be producing goods for different product markets.

Sales shares are given by,

$$s_i^y = \frac{\xi_{y,i}\left(\frac{\xi_{n,i}}{\xi_{y,i}}\right)^{\frac{\sigma-1}{\eta+\sigma}}(z_i)^{\frac{(\sigma-1)(1+\eta)}{\eta+\sigma}}\left(\frac{\mu_i^n}{\mu_i^y}\right)^{\frac{(\sigma-1)\eta}{\eta+\sigma}}\left(\upsilon_{n,\ell(i)}\right)^{\frac{(\sigma-1)(1+\eta)}{(1+\theta)(\eta+\sigma)}}\left(S_{\ell(i)}^n\right)^{\frac{(\sigma-1)(\theta-\eta)}{(1+\theta)(\eta+\sigma)}}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=\kappa(i)\}}\xi_{y,i}\left(\frac{\xi_{n,j}}{\xi_{y,j}}\right)^{\frac{\sigma-1}{\eta+\sigma}}(z_j)^{\frac{(\sigma-1)(1+\eta)}{\eta+\sigma}}\left(\frac{\mu_j^n}{\mu_j^y}\right)^{\frac{(\sigma-1)\eta}{\eta+\sigma}}\left(\upsilon_{n,\ell(j)}\right)^{\frac{(\sigma-1)(1+\eta)}{(1+\theta)(\eta+\sigma)}}\left(S_{\ell(j)}^n\right)^{\frac{(\sigma-1)(\theta-\eta)}{(1+\theta)(\eta+\sigma)}}} \tag{73}$$

As before, since the denominator sums across all firm $j \in \mathcal{M}$ that share the product market with firm $i$ product market aggregates cancel out. Labor market aggregates instead do not cancel from our expression since firms in a product market could be operating in different labor markets.

Finally, remember that firm level market power, the ratio of markups to markdowns, are only function of sales and payroll shares. So given values for market level shares $\{\{S_k^y\}_{k\in\mathcal{K}}, \{S_l^n\}_{l\in\mathcal{L}}\}$, we can use the recursion defined by the equations (72) and (73), to solved for the sales and payroll shares at the firm level.

### Prices as functions of sales and payroll shares

Before describing the algorithm we use to solved for the equilibrium is convenient to express firm level prices and wages as functions of shares only. We start with the labor supply equation for firm $i$. We use that $s_i^n = \xi_{n,i}\left(\frac{w_i}{W_{\ell(i)}}\right)^{1+\eta}$ and $s_i^n = \upsilon_{n,\ell(i)}\left(\frac{W_{\ell(i)}}{W}\right)^{1+\theta}$ to express $n_i$ as,

$$n_i = \tilde{n}_i N,$$

where we defined,

$$\tilde{n}_i \equiv \left(\xi_{n,i}\left(s_i^n\right)^{\eta}\right)^{\frac{1}{1+\eta}}\left(\upsilon_{n,\ell(i)}\left(S_{\ell(i)}^n\right)^{\theta}\right)^{\frac{1}{\theta+1}}.$$

Note that $\tilde{n}_i$ is only a function sale and payroll shares.

Using the production function of the intermediate good producer we can write firm level output as,

$$y_i = \tilde{y}_i N,$$

where $\tilde{y}_i \equiv z_i \tilde{n}_i$ is again only a function of shares. Therefore, market level and aggregate output are

simply,

$$Y_k = \left[ \sum_{\{i \in \mathcal{M}: \kappa(i)=k\}} \xi_{y,i}^{\frac{1}{\sigma}} \tilde{y}_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} N = \tilde{Y}_k N,$$

$$Y = \left[ \sum_{k \in \mathcal{K}} \upsilon_{y,k}^{\frac{1}{\epsilon}} \tilde{Y}_k^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} N = \tilde{Y} N.$$

Finally, we can use the inverse demand equation to express prices as functions of tilde variables which only depend on sale and payroll shares,

$$p_i = \left( \frac{1}{\xi_{y,i}} \frac{y_i}{Y_{\kappa(i)}} \right)^{-\frac{1}{\sigma}} \left( \frac{1}{\upsilon_{y,\kappa(i)}} \frac{Y_{\kappa(i)}}{Y} \right)^{-\frac{1}{\epsilon}} P = \left( \frac{1}{\xi_{y,i}} \frac{\tilde{y}_i}{\tilde{Y}_{\kappa(i)}} \right)^{-\frac{1}{\sigma}} \left( \frac{1}{\upsilon_{y,\kappa(i)}} \frac{\tilde{Y}_{\kappa(i)}}{\tilde{Y}} \right)^{-\frac{1}{\epsilon}} \equiv \tilde{p}_i, \tag{74}$$

where the last equality uses that aggregate consumption is the numeraire of the economy.

Firm level wages can be computed through the optimal pricing equation of the firm,

$$w_i = z_i \frac{\mu_i^n}{\mu_i^y} \tilde{p}_i. \tag{75}$$

Since firm market power only depends on firm level share, and we showed prices can be written as functions of shares as well, wages are itself a function of shares only. Equation 66 can then be used to compute for each labor market the market level wage index,$\{W_l\}_{l \in \mathcal{L}}$ and the aggregate wage index $\{W\}$.

### Algorithm: Autarky Equilibrium

A nested fixed point algorithm can be used to solved for the autarky equilibrium. In this section we describe the procedure. We use $t^i$ and $t^o$ to denote iterations for the inner and the outer nest, respectively. The outer nest start with a guess of market level payroll shares for each labor market $\left\{ S_{l,t^o}^n \right\}_{l \in \mathcal{L}}$ and a guess of market level sales shares for each product market $\left\{ S_{k,t^o}^y \right\}_{k \in \mathcal{K}}$. Given the parameters of the model and the market level shares, $\left\{ S_{l,t^o}^n \right\}_{l \in \mathcal{L}}, \left\{ S_{k,t^o}^y \right\}_{k \in \mathcal{K}}$, we construct the firm level shares that correspond to the efficient allocation for the given values of the market level shares, and denote them as $\{s_{i,t^i}^n, s_{i,t^i}^y\}_{i \in \mathcal{M}}$. To construct these pseudo-efficient firm level shares we use equations (72) and (73), but we impose that firm level markups and markdowns are both equal to

one as in the efficient allocation.

**Step 1. Inner Nest.** We use $\{s^n_{i,t^i}, s^y_{i,t^i}\}_{i\in\mathcal{M}}$ as the initial guess. Then:

1. We construct firm level market power as,

$$\frac{1}{\mu^y_{i,t^i}} = \frac{\sigma-1}{\sigma} - \left(\frac{1}{\epsilon}-\frac{1}{\sigma}\right) s^y_{i,t^i}, \quad \text{and} \quad \frac{1}{\mu^n_{i,t^i}} = \frac{\eta+1}{\eta} + \left(\frac{1}{\theta}-\frac{1}{\eta}\right) s^n_{i,t^i},$$

   and use the right-hand side of equations (72) and (73) to update the guessed value of the firm-level shares, $\{s^n_{i,t^i+1}, s^y_{i,t^i+1}\}_{i\in\mathcal{M}}$.

2. We iterate over this procedure until the resulting shares from two consecutive iterations are indistinguishable from one another. That is, we stop when the following condition is satisfy,

$$\max\left\{\max_{j\in\mathcal{M}}\left\{\left|s^n_{j,t^i+1} - s^n_{j,t^i}\right|\right\}, \max_{j\in\mathcal{M}}\left\{\left|s^y_{j,t^i+1} - s^y_{j,t^i}\right|\right\}\right\} \leq 1e^{-08}$$

3. In practice, we find useful to move from one iteration to the next one in small steps. So the updated guess is constructed as, $s_{i,t^i+1} = (1-\psi)s_{i,t^i+1} + \psi s_{i,t^i}$. We use $\psi = 0.9$.

**Step 2. Outer Nest.** Once Step 1 is completed we end up with our initial guess for the market level shares $\left\{S^n_{l,t^o}\right\}_{l\in\mathcal{L}}, \left\{S^y_{k,t^o}\right\}_{k\in\mathcal{K}}$, and the equilibrium values of the firm level shares that are consistent with these market level shares. We denote them as, $\{s^n_{i,out}, s^y_{i,out}\}_{i\in\mathcal{M}}$. Then,

1. We use the initial guess for the market level shares and the resulting firm level shares to construct firm level prices and firm level wages using equations (74) and (75).

2. We then construct the market and aggregate level price and wage index using the firm level prices and wages and the definition of theses indexes.

3. We update the market level shares using the formulas,

$$S^n_{l,t^o+1} = \upsilon_{n,l}\left(\frac{W_{l,t^o}}{W_{t^o}}\right)^{1+\theta}, \quad \text{and} \quad S^y_{k,t^o+1} = \upsilon_{y,k}\left(\frac{P_{k,t^o}}{P_{t^o}}\right)^{1-\epsilon}$$

4. If

$$\max\left\{\max_{l\in\mathcal{L}}\left\{\left|S^n_{l,t^o+1} - S^n_{l,t^o}\right|\right\}, \max_{k\in\mathcal{K}}\left\{\left|S^y_{k,t^o+1} - S^y_{k,t^o}\right|\right\}\right\} \leq 1e^{-08},$$

we have found the equilibrium and the algorithm stops. If this condition does not hold, then we update the guess for the market level shares with $S_{t^o+1} = (1-\psi)S_{t^o+1} + \psi S_{t^o}$ and $\psi = 0.9$ and use it to initiate Step 1 and Step 2 again.

**Step 3.** After Step 1 and Step 2 are completed we end up with the market and firm level shares and all the prices and wages of the economy. The last step uses this information to construct the quantities.

Once we know the equilibrium value of the aggregate wage we can compute aggregate employment using the first order condition of the household,

$$N = \frac{1}{\bar{\varphi}} W^{\varphi}.$$

Then, from the labor supply equation we can construct firm level employment,

$$n_i = \left(\xi_{n,i} \left(s_i^n\right)^{\eta}\right)^{\frac{1}{1+\eta}} \left(\upsilon_{n,\ell(i)} \left(S_{\ell(i)}^n\right)^{\theta}\right)^{\frac{1}{\theta+1}} N,$$

and the production function give us firm level output, $y_i = z_i n_i$. We recover market level and aggregate quantities by aggregating the firm level employment and output.

# D Market aggregation of firm market power

## D.1 Market level markdown

**Proposition D.1.** When labor supply is nested CES, and firms compete for workers a la Cournot, the average wage markdown at labor market $l$ is given by:

$$\mu_l^n \equiv \left[\frac{\bar{r}_l}{\bar{w}_l}\right]^{-1} = \left[1 + \left(\gamma_l^n\right)^{-1}\right]^{-1} = \left[1 + \frac{1}{\theta} HHI_l + \frac{1}{\eta}\left(1 - HHI_l\right)\right]^{-1},$$

where $\bar{r}_l$ and $\bar{w}_l$ are market $l's$ (employment-weighted) average marginal revenue product of labor and average wage, respectively, $\left(\gamma_l^n\right)^{-1}$ is the (payroll-weighted) average inverse elasticity of firm-specific residual labor supply across firms in market $l$, and $HHI_l = \sum_{\{i \in \mathcal{M}: \ell(i) = l\}} \left(s_i^n\right)^2$ is the market's payroll Herfindahl.

*Proof.* First we show that $1+(\gamma_l^n)^{-1} = 1+\frac{1}{\theta}HHI_l + \frac{1}{\eta}(1-HHI_l)$:

$$\begin{aligned}
1+(\gamma_l^n)^{-1} &\equiv \sum_{\{i\in\mathcal{M}:\ell(i)=l\}} s_i^n \left(1+(\gamma_i^n)^{-1}\right) \\
&= \sum_{\{i\in\mathcal{M}:\ell(i)=l\}} s_i^n \left(\frac{\eta+1}{\eta} + \left(\frac{1}{\theta}-\frac{1}{\eta}\right) s_i^n\right) \\
&= \frac{\eta+1}{\eta} + \left(\frac{1}{\theta}-\frac{1}{\eta}\right) \sum_{\{i\in\mathcal{M}:\ell(i)=l\}} (s_i^n)^2 \\
&= 1+\frac{1}{\eta}(1-HHI_l) + \frac{1}{\theta}HHI_l.
\end{aligned}$$

Second we show that $\sum_{\{i\in\mathcal{M}:\ell(i)=l\}} s_i^n \left(1+(\gamma_i^n)^{-1}\right) = \frac{\bar{r}_l}{\bar{w}_l}$. To see this note that,

$$\begin{aligned}
\sum_{\{i\in\mathcal{M}:\ell(i)=l\}} s_i^n \left(1+(\gamma_i^n)^{-1}\right) &= \sum_{\{i\in\mathcal{M}:\ell(i)=l\}} s_i^n \left(\frac{1}{\mu_i^n}\right) \\
&= \sum_{\{i\in\mathcal{M}:\ell(i)=l\}} s_i^n \left(\frac{r_i}{w_i}\right) \\
&= \sum_{\{i\in\mathcal{M}:\ell(i)=l\}} \frac{w_i n_i}{\sum_{\{j\in\mathcal{M}:\ell(j)=l\}} w_j n_j} \left(\frac{r_i}{w_i}\right) \\
&= \frac{\sum_{\{i\in\mathcal{M}:\ell(i)=l\}} n_i r_i}{\sum_{\{j\in\mathcal{M}:\ell(j)=l\}} w_j n_j} \\
&= \frac{\sum_{\{i\in\mathcal{M}:\ell(i)=l\}} \frac{n_i}{\sum_{\{h\in\mathcal{M}:\ell(h)=l\}} n_h} r_i}{\sum_{\{j\in\mathcal{M}:\ell(j)=l\}} \frac{n_j}{\sum_{\{h\in\mathcal{M}:\ell(h)=l\}} n_h} w_j} \\
&= \frac{\bar{r}_l}{\bar{w}_l}.
\end{aligned}$$

Putting the two together,

$$\frac{1}{\mu_l^n} = 1+(\gamma_l^n)^{-1} = \frac{\bar{r}_l}{\bar{w}_l}.$$

□

**Proposition D.2.** When product demand is nested CES, and firms compete a la Cournot, the average markup at product market $k$ is given by:

$$\mu_k^y \equiv \left[\frac{\bar{mc}_k}{\bar{p}_{H,k}}\right]^{-1} = \left[1-\left(\gamma_k^y\right)^{-1}\right]^{-1} = \left[1-\frac{1}{\epsilon}(1-\lambda_k)\,HHI_k^d - \frac{1}{\sigma}\left(1-(1-\lambda_k)\,HHI_k^d\right)\right]^{-1},$$

where $\bar{mc}_k$ and $\bar{p}_{Hk}$ are market $k's$ (quantity-weighted) average marginal cost and average price,

respectively, $\left(\gamma_k^y\right)^{-1}$ is the (sales-weighted) average inverse elasticity of firm-specific residual product demand across firms in market $k$, $HHI_k^d = \sum_{\{i\in\mathcal{M}:\ell(i)=l\}} \left(s_i^y\right)^2$ is the market's domestic sales Herfindahl, and $\lambda_k$ is expenditure share on imported goods.

*Proof.* First we show that $1-\left(\gamma_k^y\right)^{-1} = 1-\frac{1}{\epsilon}\left(1-\lambda_k\right)HHI_k^d - \frac{1}{\sigma}\left(1-\left(1-\lambda_k\right)HHI_k^d\right)$.:

$$\begin{aligned}
1-\left(\gamma_k^y\right)^{-1} &\equiv \sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} s_i^y\left(1-\left(\gamma_i^y\right)^{-1}\right) \\
&= \sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} s_i^y\left(\frac{1}{\mu_i^y}\right) \\
&= \sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} s_i^y\left(\frac{\sigma-1}{\sigma} - \left(\frac{1}{\epsilon}-\frac{1}{\sigma}\right)s_i^y\left(1-\lambda_k\right)\right) \\
&= 1-\frac{1}{\sigma} - \left(1-\lambda_k\right)\left(\frac{1}{\epsilon}-\frac{1}{\sigma}\right)\sum_{\{i\in\mathcal{M}:\kappa(i)=k\}}\left(s_i^y\right)^2 \\
&= 1-\frac{1}{\epsilon}\left(1-\lambda_k\right)HHI_k^d - \frac{1}{\sigma}\left(1-\left(1-\lambda_k\right)HHI_k^d\right).
\end{aligned}$$

Second we show that $\sum_{\{i\in\mathcal{M}:\ell(i)=l\}} s_i^n\left(1+\left(\gamma_i^n\right)^{-1}\right) = \frac{\bar{r}_l}{\bar{w}_l}$. To see this note that,

$$\begin{aligned}
\sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} s_i^y\left(1-\left(\gamma_i^y\right)^{-1}\right) &= \sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} s_i^y\left(\frac{1}{\mu_i^y}\right) \\
&= \sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} s_i^n\left(\frac{mc_i}{p_{Hi}}\right) \\
&= \sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} \frac{p_{Hi}y_{Hi}}{\sum_{\{j\in\mathcal{M}:\kappa(j)=k\}} p_{Hj}y_{Hj}}\frac{mc_i}{p_{Hi}} \\
&= \frac{\sum_{\{i\in\mathcal{M}:\kappa(i)=k\}} \frac{y_{Hi}}{\sum_{\{h\in\mathcal{M}:\kappa(h)=k\}} y_{Hh}} mc_i}{\sum_{\{j\in\mathcal{M}:\kappa(j)=k\}} \frac{y_{Hj}}{\sum_{\{h\in\mathcal{M}:\kappa(h)=k\}} y_{Hh}} p_{Hj}} \\
&= \frac{\bar{mc}_k}{\bar{p}_{H,k}}.
\end{aligned}$$

Putting the two together,

$$\frac{1}{\mu_l^y} \equiv 1+\left(\gamma_k^y\right)^{-1} = \frac{\bar{mc}_k}{\bar{p}_{H,k}}.$$

□

# E Estimating the elasticities of substitution

In this section we described our strategy to estimate the four elasticities of substitutions: The two elasticities in the labor market ($\eta$ and $\theta$), and the two elasticities in the product market ($\sigma$ and $\epsilon$). The strategy discussed in this note is preliminary and it is in the process of being implemented.

## E.1 Elasticities of substitution in Labor Markets

Amiti et al. (2019) provide direct empirical estimate's of firms' price responses to changes in competitor prices to answer the question: *How strong are strategic complementarities in price setting across firms?.* In this note we show how to adapt their methodology to study strategic complementarities in wage setting across firms.

**Notation.** We use $i$, $j$ and $k$ to denote individual firms, and $l$ to denote labor markets. $M_l$ represents the total number of firms in labor market $l$, while $\mathcal{L}$ represents the set of labor markets. We use $t$ to denote time. We will derive the equations for a particular labor market $l$, so in most cases we won't index variables by $l$ unless we have to.

We denote with $\mathbf{w}_t = (w_{1t}, \ldots, w_{M_l,t})'$ the vector of wages paid by firms in labor market $l$, and use the notation $\mathbf{w}_{-it} = \{w_{jt}\}_{j \neq i}$ to denote the vector of wages that exclude firm's $i$. Hence, $\mathbf{w_t} = (w_{it}, \mathbf{w}_{-it})$.

### E.1.1 Deriving our main equation.

We consider an arbitrary invertible supply system $n_i = n_i(\mathbf{w}, \xi)$ for $i \in \{1, 2, \ldots, M_l\}$ which constitutes a one-to one mapping between any vector of wages $\mathbf{w}_t$ and a corresponding vector of quantities supplied $\mathbf{n}_t$, given the vector of supply shifters $\xi_t = \{\xi_{tj}\}_{j=1}^{M^l}$. The supply shifters summarize all variables that move the quantity supplied for a given wage vector. The invertibility assumption rules out the case of perfect substitutes, where multiple allocations of quantities across individuals are consistent with the same common wage.

Under these assumptions on supply and market structure, we prove the existence of a markdown function, which characterizes the firm's optimal wage-setting strategy in a given labor market

equilibrium.

**Proposition E.1.** For any given invertible supply system, there exists a markdown function $\mu_{it} = \mathcal{M}_i(w_{it}, \mathbf{w}_{-it}; \xi_t)$ such that the firm's static profit minimization wage $\tilde{w}_{it}$ is the solution to the following fixed point equation, for any given wage vector of the competitors $\mathbf{w}_{-it}$:

$$\tilde{w}_{it} = mrpl_{it} + \mathcal{M}_i(w_{it}, \mathbf{w}_{-it}, \xi_t).$$

*Proof.* Consider the profit maximization problem of the firm

$$\max_n \{p(y)\, y - w(n)\, n | \text{s.t. } y = f(n)\},$$

where the firm chooses the amount of labor $n$ and inputs $\mathbf{x}$ to maximize profits. The firm internalize the labor supply $w(n)$ and the demand for its products $p(y)$. This case accommodates cases where the firm has market power in product markets as well. The first-order condition for labor are given by,

$$[p'(y)\, y + p(y)] \frac{\partial y}{\partial n} = w(n) \left[\frac{\gamma(\mathbf{w}, \xi) + 1}{\gamma(\mathbf{w}, \xi)}\right]. \tag{76}$$

The left-hand side of this equation represents the marginal revenue product of labor ($MRPL$) of the firm. Note that $MRPL$ incorporates the effect that more output decreases the price of the firm. From (76) we obtain the expression for the optimal markdown of the firm. Defining the markdown as $\mu_i \equiv \log w - \log MRPL$, we get,

$$\mu_i = \log \frac{\gamma(\mathbf{w}, \xi)}{\gamma(\mathbf{w}, \xi) + 1},$$

where $\varepsilon(\mathbf{w}, \xi)$ is the perceived elasticity of supply. Therefore, the optimal wage of the firm solves the following fixed point equation:

$$w_i = mrpl_i + \mu_i.$$

□

Proposition E.1 does not require that competitor prices are equilibrium outcomes, as ti holds for any possible vector $\mathbf{w}_{-it}$, and corresponds to the firm's best response schedule. The full market

equilibrium is achieved when equation (76) holds for every firm in the market, that is all firms are on their response schedules.

From now on we focus on equation (76) at the equilibrium,

$$w_{it} = mrpl_{it} + \mathcal{M}_i\left(w_{it}, \mathbf{w}_{-it}, \xi_t\right). \tag{77}$$

### E.1.2 Wage change decomposition

Totally differentiating (in time) the previous expression around the equilibrium point give us the following equation for the change in wages,

$$\mathrm{d}w_{it} = \frac{1}{1+\Gamma_{it}}\mathrm{d}mrpl_{it} + \frac{\Gamma_{-it}}{1+\Gamma_{it}}\mathrm{d}w_{-it} + \varepsilon_{it}, \tag{78}$$

where $\Gamma_{it}$ and $\Gamma_{-it}$ where defined as:

$$\Gamma_{it} \equiv -\frac{\partial \mathcal{M}_i\left(\mathbf{w}_t, \xi_t\right)}{\partial w_{it}} \quad \text{and} \quad \Gamma_{-it} \equiv \sum_{\{j\neq i, j\in l\}} \frac{\partial \mathcal{M}_i\left(\mathbf{w}_t, \xi_t\right)}{\partial w_{jt}}, \tag{79}$$

and represent the own and (cumulative) competitor markdown elasticity, respectively, measuring the slope of the optimal markdown function. 'The term $\varepsilon_i$ is firm's $i$ effective labor supply shock and it is given by,

$$\varepsilon_{it} \equiv \frac{1}{1+\Gamma_{it}} \sum_{j=1}^{M_l} \frac{\partial \mathcal{M}_i\left(\mathbf{w}_t, \xi_t\right)}{\partial \xi_{jt}} \mathrm{d}\xi_{jt} \tag{80}$$

Finally, $\mathrm{d}w_{-it}$ is an index of competitor wage changes:

$$\mathrm{d}w_{-it} \equiv \sum_{\{j\neq i, j\in l\}} \omega_{ijt}\mathrm{d}w_{jt}, \quad \text{where} \quad \omega_{ijt} \equiv \frac{\partial \mathcal{M}_i\left(\mathbf{w}_t, \xi_t\right)/\partial w_{jt}}{\sum_{\{k\neq i, k\in l\}} \partial \mathcal{M}_i\left(\mathbf{w}_t, \xi_t\right)/\partial w_{kt}}. \tag{81}$$

Our main equation is then,

$$\mathrm{d}w_{it} = \alpha_{it}\mathrm{d}mrpl_{it} + \gamma_{it}\mathrm{d}w_{-it} + \varepsilon_{it}, \tag{82}$$

which decomposes firm $i$'s wage change into responses to its own marginal revenue product of labor, to its competitor's wage change, and to the supply shifters captured by the residual. In practice,

however, we can estimate,[48]

$$\Delta w_{it} = \alpha \Delta mrpl_{it} + \gamma \Delta w_{-it} + \varepsilon_{it}, \tag{83}$$

where $\Delta w_{it} = w_{i,t+1} - w_{it}$. The parameter $\alpha$ captures the mean pass-through of shocks from the marginal-revenue product of labor to wages. The parameter $\gamma$ captures the degree of strategic complementarity in wage setting. These two parameters are shaped by the markdown elasticities: a higher own-markdown elasticity reduces the own pass-through, as markdowns are more accommodative of shocks, while a higher competitor markdown elasticity increases the strategic complementarity elasticity.

In order to empirically estimate the coefficients in the theoretical wage decomposition (83), we need to measure the competitor price index $\mathbf{d}w_{-it}$ and the marginal revenue product of labor of the firm. We now provide conditions under which both can be measure from data.

### E.1.3 Constructing the equation regressors

**Identifying $dw_{-it}$.** Let $W_{lt}$ denote the log payroll function of labor market $l$, which quantifies the cost of purchasing one unit of aggregate labor in a given labor market. Formally, it is defined as,

$$W_t \equiv \log \max_{n_{it}} \left\{ \sum_{i=1}^{M_l} \exp w_{it} + n_{it} \middle| U\left(\{n_{it}\}, N_t, \xi_t\right) = 1, N_t = 1 \right\},$$

where $U(\cdot)$ is the preference aggregator which defines the unit of market-level employment $N_t$.

**Definition E.1.** We say that $W_{lt}$ is a sufficient statistic for a firm's competitors wage if we can write the labor supply face by the firm as $n_{it} = n_i\left(w_{it}, W_t, \xi_t\right)$.

**Proposition E.2.** If $W_t$ is a sufficient statistic for a firm's competitors wage then the weights in the competitor wage index are proportional to the competitor payroll shares $s^n_{jt}$,for $j \neq i$, and given by $\omega_{ijt} \equiv \frac{s^n_{jt}}{1-s^n_{it}}$. Therefore, the index of competitor price changes simplifies to:

$$\mathbf{d}w_{-it} \equiv \sum_{\{j \neq i, j \in l\}} \frac{s^n_{jt}}{1 - s^n_{it}} \mathrm{d}w_{jt}.$$

[48]The estimated parameter $\gamma$ is a measure of strategic complementarities in wage setting. A positive value for $\gamma$ is evidence of oligopsonistic labor markets.

Under the strong assumption that the perceived supply elasticity is a function of the wage of the firm relative to the market payroll function, $\gamma_{it} = \gamma_i (w_{it} - W_t; \xi_t)$, the two markdown elasticities are equal,

$$\Gamma_{-it} = \Gamma_{it}.$$

*Proof.* If $n_i = n_i (w_i, z; \xi)$ then we can follow the same steps as in Proposition E.1, we can show there exists a markdown function:

$$\mu_i = \mathcal{M}_i (w_i, z; \xi) \equiv -\log \frac{\gamma_i (w_i, z; \xi) + 1}{\gamma_i (w_i, z; \xi)},$$

such that the profit maximizing wage of the firm solves $w_i = mrpl_i + \mu_i$. Now using the definition of the competitor wage change index and the properties of the log payroll function $W = W (w, \xi)$, we have:

$$\begin{aligned}
\omega_{ijt} &= \frac{\partial \mathcal{M}_i (\mathbf{w}_t, \xi_t) / \partial w_{jt}}{\sum_{\{k \neq i, k \in l\}} \partial \mathcal{M}_i (\mathbf{w}_t, \xi_t) / \partial w_{kt}} \\
&= \frac{(\partial \mathcal{M}_i (w_i, W; \xi) / \partial W) \times (\partial W / \partial w_j)}{\sum_{\{k \neq i, k \in l\}} (\partial \mathcal{M}_i (w_i, W; \xi) / \partial W) \times (\partial W / \partial w_k)} \\
&= \frac{(\partial \mathcal{M}_i (w_i, W; \xi) / \partial W) \times s_j^n}{\sum_{\{k \neq i, k \in l\}} (\partial \mathcal{M}_i (w_i, W; \xi) / \partial W) \times s_k^n} \\
&= \frac{s_j^n}{\sum_{\{k \neq i, k \in l\}} \times s_k^n} \\
&= \frac{s_j^n}{1 - s_i^n}.
\end{aligned}$$

where we use the envelope condition to conclude that,

$$\frac{\partial W}{\partial w_j} = \frac{W_j N_j}{\sum_{\{k \in l\}} W_k N_k} = s_j^n.$$

If a stronger condition $\gamma_i = \gamma_i (w_i - z; \xi)$ is satisfied, then:

$$\mu_i = \mathcal{M}_i (w_i - W; \xi) \equiv \log \frac{\gamma_i (w_i - W; \xi)}{\gamma_i (w_i - W; \xi) - 1},$$

and using the definition of $\Gamma_i$ and $\Gamma_{-i}$ we have:

$$\Gamma_i = -\frac{\partial \mathcal{M}_i\left(\mathbf{w}_t - W, \xi_t\right)}{\partial w_i}\left(1 - \frac{\partial W}{\partial w_{it}}\right) = -\frac{\partial \mathcal{M}_i\left(\mathbf{w}_t - W, \xi_t\right)}{\partial w_i}\left(1 - s_i^n\right)$$
$$\Gamma_{-i} = -\frac{\partial \mathcal{M}_i\left(w_i - W, \xi_t\right)}{\partial w_{it}} \sum_{\{j \neq i, j \in l\}} \left(s_j^n\right) = -\frac{\partial \mathcal{M}_i\left(w_i - W, \xi_t\right)}{\partial w_{it}}\left(1 - s_i^n\right).$$

□

Definition E.1 may seem strange at first but it holds exactly for the nested-CES structure as it show in the example below.

**Nested CES.** In the oligoponistic model of Berger et al. (2022a) we have:

$$N_i = \xi_i \upsilon_l \frac{N}{W^\theta}\left(\frac{W_i}{\tilde{W}_l}\right)^\eta \tilde{W}_l^\theta,$$

where $\tilde{W}_l$ is the payroll function and therefore $W_l = \log \tilde{W}_l$. Then,

$$n_i = \log\left(\xi_i \upsilon_l \frac{N}{W^\theta}\right) + \eta w_i + (\theta - \eta) W.$$

Note then that $W$is a sufficient statistic for a firm's competitors wage. Furthermore, the inverse supply curve is given by,

$$w_i = \left(\frac{1}{\xi_i}\frac{N_i}{N_l}\right)^{\frac{1}{\eta}}\left(\frac{1}{\upsilon_l}\frac{N_l}{N}\right)^{\frac{1}{\theta}} W,$$

and so the labor supply elasticity is,

$$\gamma_i \equiv \left[\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n\right]^{-1},$$

which implies that $\gamma_i = \gamma_i\left(w_i - W; \xi\right)$ since,

$$s_i^n \equiv \frac{W_i N_i}{\sum W_j N_j} = \frac{\xi_i \left(W_i\right)^{1+\eta}}{\sum \xi_j \left(W_j\right)^{1+\eta}} = \xi_i e^{(1+\eta)(w_i - W)}.$$

Finally, we have

$$\mu_i = \log \frac{\gamma_i}{\gamma_i + 1} = -\log\left(\frac{\eta + 1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n\right)$$

So the model implied markdown elasticities is:

$$\begin{aligned}\Gamma_i &\equiv -\frac{\partial \mu_i}{\partial w_i} \\ &\equiv \frac{1}{\frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n} \left(\frac{1}{\theta} - \frac{1}{\eta}\right) \frac{\partial s_i^n}{\partial w_i} \\ &\equiv \frac{1}{\frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n} \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n (1+\eta)(1-s_i^n).\end{aligned}$$

Rearranging terms:

$$\Gamma_i = \frac{\left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n (1+\eta)(1-s_i^n)}{\frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n}.$$

Also,

$$\begin{aligned}\frac{\partial \mu_i}{\partial w_j} &= -\frac{1}{\frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n} \left(\frac{1}{\theta} - \frac{1}{\eta}\right) \frac{\partial s_i^n}{\partial w_j} \\ \implies \Gamma_{-i} &= \frac{1}{\frac{\eta+1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n} \left(\frac{1}{\theta} - \frac{1}{\eta}\right) s_i^n (1+\eta)(1-s_i^n)\end{aligned}$$

The main additional insight from this example is that $\Gamma_i \approx 0$ for the very small firms with $s_i^n \approx 0$. Such small firms behave nearly as constant-markup monopolistic competitors, with a complete own marginal product pass-through and no strategic complementarities. In contrast, firms with positive market shares have $\Gamma_{-it} = \Gamma_{it} > 0$ and hence exhibit incomplete pass-through and positive strategic complementarities.

Since proposition E.2 holds true in our model we can recover $dw_{-it}$ directly from our data. Furthermore, since $\Gamma_i = \Gamma_{-i}$ Proposition E.2 offers a useful way to empirically test the implication of its assumptions: the condition on markdown elasticities implies that the two coefficients in the wage decomposition sum to one:

$$\alpha_{it} + \gamma_{it} = 1.$$

We do not impose this condition in our estimation, but instead test it empirically.

**Identifying** $d\ mrplw_{it}$**.** In order to identify the firm's marginal revenue product of labor we need to impose assumptions on the firm's production function and on firm's behavior.[49]

[49] Amiti, Itskhoki and Konings (2019) impose a functional form for the marginal cost of the firm. This restriction allows them to recover changes in the marginal cost from changes in the total variable cost relative to changes in the

**Assumption E.1.** Let $\theta_i^X$ denote firm $i's$ output elasticity with respect to input $X$. The output elasticity $\theta_i^X$ is firm specific but **time-invariant,**

$$\theta_i^X \equiv \frac{\partial \log Y_{it}}{\partial \log X_{it}}.$$

**Assumption E.2.** There is at least one input ($V_{it}$), different from labor, that is "flexible", i.e. $V_{it}$ is a variable input for which the firm does not have market power.

**Proposition E.3.** Under assumptions E.1 and E.2 firm $i's$ log marginal revenue product of labor is given by,

$$mrpl_{it} = \log\left(P_{vt}V_{it}\right) - n_{it} + \log\left(\frac{\theta_i^n}{\theta_i^v}\right),$$

where $P_{vt}$ is the price of input $V$, and $n_{it}$ is firm $i's$ log employment. Furthermore,

$$d\ mrpl_{it} = d\log\left(P_{vt}V_{it}\right) - dn_{it}.$$

*Proof.* Profit maximization of the firm implies:

$$\mu_i^y + p_i + \log\frac{\partial Y_i}{\partial N_i} = w - \mu_i^n.$$

At the same time, static cost minimization of the firm implies:

$$p_v = mc_i + \log\theta_i^v + \left(y_i - v_i\right), \qquad \text{and } w_i - \mu_i^n = mc_i + \log\theta_i^n + \left(y_i - n_i\right).$$

Combining these three equations we get:

$$mrpl_i = p_v + v_i - n_i + \log\frac{\theta_i^n}{\theta_i^v}.$$

It follows that:

$$d\ mrpl_{it} = d\log\left(P_{vt}V_{it}\right) - dn_{it}.$$

□

total amount of output produced. The variable cost is taken from the data while the output is constructed from firm revenues deflated by a firm price index.

Proposition E.3 tell us that we can recover changes in marginal revenue product of labor from changes in employment and firms total payments to a flexible input, both directly observed in our data.[50]

### E.1.4 Instrumental Variables

Equation (83) has two major empirical challenge left: (i) endogeneity of the competitors wages, which are determined simultaneously with the wage of the firm in equilibrium of the wage-setting game; (ii) measurement error in the marginal revenue product of labor.

Amiti et al. (2019) exploits rare features of their data to construct instruments to address these issues in the context of price setting behavior. Their identification strategy exploits the idiosyncratic variation in firms' marginal costs, which arises as firms, even within the same industry, source their intermediate inputs from different countries and suppliers. Then, using information on firm import prices they construct a proxy for each firms marginal cost, and construct instruments based on these proxy variables. Our identification strategy mimics their construction of instruments, but here we instrument for the marginal revenue product of labor instead of the marginal cost. We exploits the idiosyncratic variation in firms' marginal revenue product of labor, which arises as firms, even within the same labor market, produce goods for different product markets.

**Baseline instruments**. We delay the discussion of the instrument for now and denote it with $\mathrm{d}mrpl^*_{it}$. To address the endogeneity of the competitor wage index $\Delta w_{-it}$, we construct instruments that proxy for the marginal revenue product of labor of the different competitors faced by Ausralian firms. We construct a weighted average of the proxied $mrpl$ of firm $i$'s competitors using the weights defined in Proposition E.2:

$$\Delta mrpl^*_{-it} \equiv \sum_{j \neq i} \frac{s^n_{jt}}{1 - s^n_{it}} \Delta mrpl^*_{jt}.$$

[50] There are some similarities between our assumptions E.1 and E.2 and the assumptions imposed by the literature that estimates firm-level markups/markdowns. However, note that we don't have to fully parameterized the production function, nor we need to specify a process for firm-level productivity. This is because we care about changes in $mrpl$ instead of its level, and therefore we don't have to go through the process of estimating the parameters $\theta^n_i$ and $\theta^v_i$.

### E.1.5 Recovering the elasticities.

The parameters estimated in equation (83), $\alpha$ and $\gamma$, are related to the elasticity of substitution within ($\eta$) and across labor markets ($\theta$). We have the following relationships:[51]

$$\alpha_i = \frac{1}{1+\Gamma_{it}}, \quad \text{and} \quad \Gamma_{-it} = \Gamma_{it} = \frac{\left(\frac{1}{\theta}-\frac{1}{\eta}\right) s_i^n \left(1+\eta\right)\left(1-s_i^n\right)}{\frac{\eta+1}{\eta}+\left(\frac{1}{\theta}-\frac{1}{\eta}\right) s_i^n}.$$

Because our model implies a relationship between $\alpha$ and $\gamma$ we have one estimate to recover two parameters $(\eta, \theta)$. We fixed a value for $\theta$ and chose $\eta$ to match our estimate of $\alpha$ using BLADE data to construct the average value for $\Gamma_i$.

Our choice of fixing $\theta$ instead of $\eta$ is because papers that have estimated a similar labor supply structure have found similar values for $\theta$, but there is a larger variation in the point estimates of $\eta$ (see for example Berger et al., 2022a; Felix, 2021).

## E.2 Elasticity of substitution in Product Markets

To recover the two parameters in the product markets, we note that our model implies the following non-linear relationship between the share of firm's revenue that is pay to workers, and the sales and payroll shares of the firm for non-exporters:

$$\frac{w_i n_i}{p_{Hi} y_{Hi}} = \frac{\mu_i^n}{\mu_i^y} = \frac{\frac{\sigma-1}{\sigma}}{\frac{\eta+1}{\eta}+\left(\frac{1}{\theta}-\frac{1}{\eta}\right) s_i^n} - \frac{\left(\frac{1}{\epsilon}-\frac{1}{\sigma}\right) s_i^y}{\frac{\eta+1}{\eta}+\left(\frac{1}{\theta}-\frac{1}{\eta}\right) s_i^n}.$$

Given knowledge of the parameters $\eta$ and $\theta$ we can use this relationship in the data to recover $\sigma$ and $\epsilon$.

[51] We do not impose in the estimation the restriction $\alpha + \gamma = 1$. Therefore, this is a testable implication and provides evidence on whether our theoretical model is a good description of Australia's labor markets.